

find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ if z is defined by

$$y^2 z e^{x+y} - \sin(xyz) = 0.$$

1. SOLUTION

Define

$$F(x, y, z) := y^2 z e^{x+y} - \sin(xyz).$$

By applying the implicit derivation formula

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}, \quad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}},$$

we obtain

$$\begin{aligned} \frac{\partial z}{\partial x} &= z \frac{ye^{x+y} - \cos(xyz)}{x \cos(xyz) - ye^{x+y}}, \\ \frac{\partial z}{\partial y} &= \frac{z[y(y+2)e^{x+y} - x \cos(xyz)]}{y[x \cos(xyz) - ye^{x+y}]}. \end{aligned}$$