Let $f(x, y) = x - y^2$.

(a) find the gradient vector $\nabla f(3, -1)$;
(b) write down the tangent line to the level curve $f(x, y) = 2$ at the point $(3, -1)$;
(c) sketch the level curve, the tangent line and the gradient vector.

1. SOLUTION

(a) $\nabla f(3, -1) = (1, 2)$.
(b) The tangent line has equation

$$\nabla f(3, -1) \cdot (x - 3, y + 1) = 0$$

which leads to

$$x + 2y = 1\,.$$