

Find local minimum, maximum and saddle points of the function

$$f(x, y) = y \sin x .$$

1. SOLUTION

We should solve the system

$$\begin{cases} y \cos x = 0 \\ \sin x = 0 \end{cases}$$

The 2nd equation implies $x = k\pi$, $k \in \mathbb{Z}$ (that is k is any integer). If $x = k\pi$, $\cos x$ is never 0. Then by simplifying, the 1st equation leads to $y = 0$. Thus, $(k\pi, 0)$, $k \in \mathbb{Z}$ are infinitely many stationary points on the x -axis. The inspection of the Hessian matrix tells us that those are all saddle points.