

Evaluate the integral

$$\iint_R \sin(x+y) dA,$$

where R is the square with vertices $(0, 0)$, $(2, 3)$, $(5, 1)$, $(3, -2)$ by using the transformation

$$x = 2u + 3v,$$

$$y = 3u - 2v.$$

1. SOLUTION

The modulus of the Jacobian of the (linear) is 13, and R is the image of the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$. We get

$$\iint_R \sin(x+y) dA = 13 \int_0^1 \int_0^1 \sin(5u+v) dudv = \frac{13}{5}(\sin 1 - \sin 6 + \sin 5).$$