

Solve the initial value problem

$$\begin{cases} xy' + 3y + 2x = 3x^2, \\ y(1) = 1. \end{cases}$$

1. SOLUTION

The 1st step is to put the equation in the standard form obtaining

$$y' + \frac{3}{x}y = 3x - 2.$$

Then

$$A(x) = 3 \int \frac{dx}{x} = 3 \ln |x| = 3 \ln x,$$

just by looking at the initial condition.

$$\begin{aligned} y(x) &= e^{-3 \ln x} \left(c + \int e^{3 \ln x} (3x - 2) dx \right) \\ &= \frac{1}{x^3} \left(c + \int (3x^4 - 2x^3) dx \right) \\ &= \frac{c}{x^3} + \frac{3}{5}x^2 - \frac{1}{2}x \\ &= \frac{9}{10x^3} + \frac{3}{5}x^2 - \frac{1}{2}x, \end{aligned}$$

after imposing the initial condition.