

Solve the initial value problem

$$\begin{cases} y'' + 9y = 27, \\ y(0) = 4, y'(0) = 6. \end{cases}$$

1. SOLUTION

The characteristic equation of the associated homogeneous differential equation is

$$\lambda^2 + 9 = 0$$

whose roots are $\lambda_{\pm} = \pm 3i$. In this case the general solution of the homogeneous differential equation is

$$c_1 \cos 3x + c_2 \sin 3x.$$

The particular solution can be recovered by applying the method of undetermined constants, which leads in this case

$$y_p(x) = A.$$

After putting y_p inside the equation, we obtain $9A = 27$ which means $A = 3$. Thus the general integral of our equation is

$$y(x) = c_1 \cos 3x + c_2 \sin 3x + 3,$$

which leads to

$$y(x) = \cos 3x + 2 \sin 3x + 3$$

after imposing the initial conditions.