Solve the linear system

$$A\mathbf{x} = \mathbf{b}\,,$$

where

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -3 & -1 & 1 \\ 4 & 0 & -2 & 3 \end{pmatrix}\,, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}\,.$$

1. SOLUTION

After using the 2nd equation we obtain

$$B\mathbf{y} = \mathbf{c}\,,$$

where

$$B = \begin{pmatrix} 1 & -1 & 0 \\ -3 & -1 & 1 \\ 0 & -2 & 3 \end{pmatrix}\,, \qquad \mathbf{c} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}\,.$$

By putting

$$C_y = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ -3 & -2 & 3 \end{pmatrix}\,, \quad C_z = \begin{pmatrix} 1 & 0 & 0 \\ -3 & -1 & 1 \\ 0 & -3 & 3 \end{pmatrix}\,, \quad C_t = \begin{pmatrix} 1 & -1 & 0 \\ -3 & -1 & -1 \\ 0 & -2 & -3 \end{pmatrix}\,,$$

we get

$$x = 1\,,\ y = \frac{\det C_y}{\det C} = 0\,,\ z = \frac{\det C_z}{\det C} = 0\,,\ t = \frac{\det C_t}{\det C} = -1\,.$$