

Let  $f : \mathbb{R}^2 \mapsto \mathbb{R}^2$  be the linear function given by

$$u = -x + y,$$

$$v = x - 2y.$$

Check for the invertibility of  $f$  and, if it is so, determine its inverse.

1. SOLUTION

The matrix associated to  $f$  is

$$A = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix}.$$

and  $f$  is invertible if and only if  $A$  is invertible. As  $\det A = 1$ ,  $f$  is invertible. As the inverse  $A^{-1}$  of  $A$  is

$$A^{-1} = \frac{1}{\det A} \operatorname{cof}(A)^t = \begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix},$$

we obtain

$$x = -2u - v,$$

$$y = -u - v.$$