

Let $f : \mathbb{R}^2 \mapsto \mathbb{R}^2$ be the linear function given by

$$\begin{aligned}u &= x - y, \\v &= x + 2y.\end{aligned}$$

Check for the invertibility of f and, if it is so, determine its inverse. The matrix associated to f is

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}.$$

and f is invertible if and only if A is invertible. As $\det A = 3$, f is invertible. As the inverse A^{-1} of A is

$$A^{-1} = \frac{1}{\det A} \operatorname{cof}(A)^t = \begin{pmatrix} 2/3 & 1/3 \\ -1/3 & 1/3 \end{pmatrix},$$

we obtain

$$\begin{aligned}x &= 2/3u + 1/3v, \\y &= -1/3u + 1/3v.\end{aligned}$$