

Consider the following matrix

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 4 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Find the matrix U such that $U^{-1}AU = D$, where D is the diagonal matrix having on the main diagonal the eigenvalues of A .

1. SOLUTION

Consider the submatrix

$$B = \begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix}.$$

Its eigenvalues are $1, -3$. One eigenvector for $\Lambda = 1$ and for $\Lambda = -3$ is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, respectively. Then

$$U = \begin{pmatrix} 1 & 1 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Notice that in order to diagonalize A , it is enough to diagonalize the submatrix B .