

Consider the following matrix

$$A = \begin{pmatrix} -1 & 4 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

Find the matrix U such that $U^{-1}AU = D$, where D is the diagonal matrix having on the main diagonal the eigenvalues of A .

1. SOLUTION

Consider the submatrix

$$B = \begin{pmatrix} -1 & 4 \\ 1 & -1 \end{pmatrix}.$$

Its eigenvalues are $1, -3$. One eigenvector for $l = 1$ and for $l = -3$ is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, respectively. Then

$$U = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Notice that in order to diagonalize A , it is enough to diagonalize the submatrix B .