Let $f : \mathbb{R}^4 \mapsto \mathbb{R}^2$ be the linear function given by

$$u = x + y - z + 2t\,,$$
$$v = -2x - 2y + 2z - 4t\,.$$

Describe:

(a) the null–set N_f of f;
(b) the image R_f of f;
(c) verify the formula

$$\dim(N_f) + \dim(R_f) = \dim(D_f)\,,$$

where $D_f = \mathbb{R}^4$ is the domain of f.

1. SOLUTION

(a): It is enough to solve $x+y-z+2t = 0$. Then for general $a, b, c \in \mathbb{R}$, each elment in N_f has the form

$$\mathbf{x} = a \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix}\,.$$

This means that $\dim(R_f) = 3$.
(b): In the (u, v)–space we get $v = -2u$. This means $\dim(R_f) = 1$.
(c): It follows by taking into account the previuos considerations and the fact that $\dim(D_f) = \dim(\mathbb{R}^4) = 4$.