MATH 172-501 Homework Solutions 09/05

Section 5.3:

8. $f(t) = \ln(1+t^2)$ and $g(x) = \int_{1}^{x} \ln(1+t^2)\,dt$,

so by FTC 1, $g'(x) = f(x) = \ln(1+x^2)$.

14. Let $u = \sqrt{x}$. Then $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$. Also, $\frac{dh}{dx} = \frac{dh}{du}\cdot\frac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz = \frac{d}{du}\int_1^{u}\frac{z^2}{z^4+1}\,dz \cdot \frac{du}{dx} =$$

$$= \frac{u^2}{u^4+1}\cdot\frac{du}{dx} = \frac{x}{x^2+1}\cdot\frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2(x^2+1)}$$

24. $$\int_1^8 x^{-2/3}\,dx = \left[\frac{x^{1/3}}{1/3}\right]_1^8 = 3\left[x^{1/3}\right]_1^8 = 3\left(8^{1/3} - 1^{1/3}\right) =$$

$$= 3(2-1) = 3$$

34. $$\int_0^3 (2\sin x - e^x)\,dx = \left[-2\cos x - e^x\right]\Big|_0^3 =$$

$$= (-2\cos 3 - e^3) - (-2-1) = 3 - 2\cos 3 - e^3$$

60. $$g(x) = \int_{1-2x}^{1+2x} t\sin t\,dt = \int_{1-2x}^{0} t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt =$$

$$= -\int_0^{1-2x} t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt \text{, so}$$

$$g'(x) = -(1-2x)\sin(1-2x)\cdot\frac{d}{dx}(1-2x) +$$

$$+ (1+2x)\sin(1+2x)\cdot\frac{d}{dx}(1+2x) =$$

$$= 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$$

62. $F(x) = \int_{\sqrt{x}}^{2x} \arctan t \, dt = \int_{\sqrt{x}}^{0} \arctan t \, dt + \int_{0}^{2x} \arctan t \, dt =$

$= -\int_{0}^{\sqrt{x}} \arctan t \, dt + \int_{0}^{2x} \arctan t \, dt$, so

$F'(x) = -\arctan(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x}) + \arctan(2x) \cdot \frac{d}{dx}(2x) =$

$= -\frac{1}{2\sqrt{x}} \cdot \arctan(\sqrt{x}) + 2\arctan(2x)$

70. a.) $\int_a^b e^{-t^2} dt = \frac{\sqrt{\pi}}{2} \cdot \frac{2}{\sqrt{\pi}} \int_a^b e^{-t^2} dt =$

$= \frac{\sqrt{\pi}}{2} \cdot \frac{2}{\sqrt{\pi}} \left[\int_a^0 e^{-t^2} dt + \int_0^b e^{-t^2} dt \right] =$

$= \frac{\sqrt{\pi}}{2} \cdot \frac{2}{\sqrt{\pi}} \left[\int_0^b e^{-t^2} dt - \int_0^a e^{-t^2} dt \right] =$

$= \frac{\sqrt{\pi}}{2} \left[\frac{2}{\sqrt{\pi}} \int_0^b e^{-t^2} dt - \frac{2}{\sqrt{\pi}} \int_0^a e^{-t^2} dt \right] =$

$= \frac{\sqrt{\pi}}{2} \left(\mathrm{erf}(b) - \mathrm{erf}(a) \right)$

b.) $y = e^{x^2} \mathrm{erf}(x) \Rightarrow y' = e^{x^2} \mathrm{erf}'(x) + 2x \underbrace{e^{x^2} \mathrm{erf}(x)}_{y} =$

$= 2xy + e^{x^2} \cdot \frac{d}{dx} \left[\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \right] =$

$= 2xy + e^{x^2} \cdot \frac{2}{\sqrt{\pi}} \cdot e^{-x^2} = 2xy + \frac{2}{\sqrt{\pi}}$

75. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^4 + \frac{i}{n} \right) \frac{1}{n} =$

$= \lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^4 + \frac{i}{n} \right) = \int_0^1 (x^4 + x) \, dx =$

$= \left[\frac{1}{5}x^5 + \frac{1}{2}x^2 \right]_0^1 = \left(\frac{1}{5} + \frac{1}{2} \right) = \frac{7}{10}$

Section 5.5:

8. Let $u = x^3$ and $du = 3x^2\,dx$. Then $x^2\,dx = \frac{1}{3}du$, so

$$\int x^2 e^{x^3}\,dx = \int e^u\left(\tfrac{1}{3}du\right) = \tfrac{1}{3}e^u + C = \tfrac{1}{3}e^{x^3} + C$$

34. Let $u = \frac{\pi}{x}$ and $du = -\frac{\pi}{x^2}\,dx$. Then $\frac{1}{x^2}dx = -\frac{1}{\pi}du$, so

$$\int \frac{\cos(\pi/x)}{x^2}\,dx = \int \cos(u)\left(-\tfrac{1}{\pi}du\right) = -\tfrac{1}{\pi}\sin(u) + C =$$

$$= -\tfrac{1}{\pi}\sin(\pi/x) + C$$

70. Let $u = (x-1)^2$ and $du = 2(x-1)\,dx$. Then $(x-1)dx = \frac{1}{2}du$, so

when $x = 0$, $u = 1$ and when $x = 2$, $u = 1$. Thus,

$$\int_0^2 (x-1)e^{(x-1)^2}\,dx = \tfrac{1}{2}\int_1^1 e^u\,du = 0 \quad \text{since}$$

the limits of integration are equal.

90. Let $u = x + c$. Then $du = dx$ and

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(u)\,du = \int_{a+c}^{b+c} f(x)\,dx \quad \text{since}$$

the 'u' and 'x' variables are arbitrary

(dummy) variables in this case: