MATH 172.501 Homework #10

Section 11.2 :

Determine if the following series converge or diverge. If they converge, find the sum.

22. $\sum_{n=1}^{\infty} \frac{5}{\pi^n} = 5 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{\pi}\right)^n$. This is a geometric series

with $|r| = \left|\frac{1}{\pi}\right| < 1$. $\therefore$ it converges.

The sum $\sum_{n=1}^{\infty} \frac{5}{\pi^n} = 5 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{1-\frac{1}{\pi}}\right) = \frac{5}{\pi - 1}$

23. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \frac{1}{4} \sum_{n=1}^{\infty} \left(-\frac{3}{4}\right)^{n-1}$, a geometric series

that converges with $|r| = \left|-\frac{3}{4}\right| < 1$.

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \frac{1}{4} \cdot \frac{1}{1-(-3/4)} = \frac{1}{7}$$

24. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n} = 3 \cdot \sum_{n=0}^{\infty} \left(-\frac{3}{2}\right)^n$, a geometric series

that diverges with $|r| = \left|-\frac{3}{2}\right| > 1$.

29. $\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$ diverges by the Divergence Test

$\because \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2+n}{1-2n} = -\frac{1}{2} \neq 0$.

30. $\sum_{k=1}^{\infty} \frac{k^2}{k^2-2k+5}$ diverges by Divergence Test with

$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k^2}{k^2-2k+5} = 1 \neq 0.$

38. $\sum_{k=0}^{\infty} (\sqrt{2})^{-k} = \sum_{k=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k$, a geometric series that

converges with $|r| = \left|\frac{1}{\sqrt{2}}\right| < 1$.

$$\sum_{k=0}^{\infty} (\sqrt{2})^{-k} = \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1}$$

43. $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \frac{2}{(n+1)(n-1)}$

Partial Fractions: $\frac{2}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1}$

$$\Rightarrow \begin{cases} A+B=0 \\ B-A=2 \end{cases} \Rightarrow \begin{cases} B=1 \\ A=-1 \end{cases}$$

$$\Rightarrow \sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} -\frac{1}{n+1} + \frac{1}{n-1}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=3}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

47. For the series $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/n+1}\right)$,

$$s_n = \sum_{i=1}^{n} \left(e^{1/i} - e^{1/i+1}\right) = \left(e^{1} - e^{1/2}\right) + \left(e^{1/2} - e^{1/3}\right) + \dots + \left(e^{1/n} - e^{1/n+1}\right)$$

$$= e - e^{1/n+1} \Rightarrow \lim_{n\to\infty} s_n = \lim_{n\to\infty} e - e^{1/n+1} = e - 1 \text{ (convergent)}$$