MATH 172-501 Homework Solutions 9/19

Section 6.3

Use the method of cylindrical shells for the following problems.

4. Rotate the region bounded by $y = x^3$, $y = 0$, $x = 1$ and $x = 2$ about the y-axis. Find the volume of the resulting solid

$$V = 2\pi \int_1^2 x \cdot x^3 \, dx = 2\pi \left[\frac{1}{5} x^5 \right]_1^2 = 2\pi \left[\frac{32-1}{5} \right]$$

$$= \frac{62}{5}\pi$$

6. Rotate the region bounded by the curves $y = 4x - x^2$ and $y = x$ about the y-axis. Find the volume of the resulting solid.

$4x - x^2 = x \iff x = 0$ or 3.

$$V = 2\pi \int_0^3 x \left[(4x - x^2) - x \right] dx = 2\pi \int_0^3 3x^2 - x^3 \, dx$$

$$= 2\pi \left[x^3 - \frac{1}{4} x^4 \right]_0^3 = 2\pi \left(27 - \frac{81}{4} \right)$$

$$= \frac{27}{2}\pi$$

10. Rotate the region bounded by the curves $y = \sqrt{x}$, $x = 0$ and $y = 2$ about the x-axis. Find the volume of the resulting solid.

$$V = 2\pi \int_0^2 y \cdot y^2 \, dy = 2\pi \left[\frac{1}{4} y^4 \right]_0^2 = 8\pi$$

14. Rotate the region bounded by the curves $x+y=4$ & $x=y^2-4y+4$ about the x-axis. Find the volume of the resulting solid.

$4-y=y^2-4y+4 \iff y=0$ or 3

$$V=2\pi\int_0^3 y\left[(4-y)-(y^2-4y+4)\right]dy$$

$$=2\pi\int_0^3 -y^3+3y^2\,dy=2\pi\left[-\tfrac{1}{4}y^4+\tfrac{3}{3}y^3\right]_0^3$$

$$=2\pi\left[27-\tfrac{81}{4}\right]=\tfrac{27}{2}\pi$$

16. Rotate the region bounded by the curves $y=4-2x$, $y=0$ and $x=0$ about the line $x=-1$. Find the volume of the resulting solid.

$$V=2\pi\int_0^2(x+1)(4-2x)\,dx$$

$$=2\pi\int_0^2 4+2x-2x^2\,dx=4\pi\int_0^2 2+x-x^2\,dx$$

$$=4\pi\left[2x+\tfrac{1}{2}x^2-\tfrac{1}{3}x^3\right]_0^2$$

$$=4\pi\left[4+2-\tfrac{8}{3}\right]=\tfrac{40}{3}\pi$$

For the following problems, compute the volumes of the resulting solids using any method

38. Rotate the region bounded by $y=-x^2+6x-8$ and $y=0$ about the x-axis. (using discs...)

$$V=\pi\int_2^4(-x^2+6x-8)^2dx=\pi\int_2^4 x^4-12x^3+52x^2-96x+64\,dx$$

$$=\pi\left[\tfrac{1}{5}x^5-3x^4+\tfrac{52}{3}x^3-48x^2+64x\right]_2^4=\tfrac{16}{15}\pi$$

40. Rotate the region bounded by $y^2 - x^2 = 1$ and $y = 2$ about the y-axis. (Using discs...)

$$y^2 - x^2 = 1 \iff x = \pm\sqrt{y^2 - 1}$$

$$V = \pi \int_1^2 \left(\sqrt{y^2-1}\right)^2 dy = \pi \int_1^2 (y^2 - 1)\, dy$$

$$= \pi \left[\frac{1}{3}y^3 - y\right]_1^2 = \pi \left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)\right]$$

$$= \frac{4}{3}\pi$$