MATH 172-501 Homework #4 09/26

Section 6.4:

4. $$\int_1^2 \cos\left(\frac{\pi x}{3}\right)dx = \frac{3}{\pi}\sin\left(\frac{\pi x}{3}\right)\Big|_1^2 = \frac{3}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{3}{\pi}\left[0\right] = 0.$$

Since the work from $x=1$ to $x=1.5$ is equal to the negative of the work from $x=1.5$ to $x=2$, the total work is zero.

Section 6.5:

6. $$f_{AVE} = \frac{1}{1-(-1)}\int_{-1}^{1} \frac{x^2}{(x^3+3)^2}dx \qquad u = x^3+3 \quad du = 3x^2\,dx$$

$$= \frac{1}{6}\int_2^4 \frac{1}{u^2}du = \frac{1}{6}\left[-\frac{1}{u}\right|_2^4 = \frac{1}{6}\left[\frac{1}{2}-\frac{1}{4}\right] = \boxed{\frac{1}{24}}$$

26. $$f_{AVE}[a,b] = \frac{1}{b-a}\int_a^b f(t)dt = \frac{1}{b-a}\left[\int_a^c f(t)dt + \int_c^b f(t)dt\right]$$

$$= \frac{1}{b-a}\left(\frac{c-a}{c-a}\right)\int_a^c f(t)dt + \frac{1}{b-a}\left(\frac{b-c}{b-c}\right)\int_c^b f(t)dt$$

$$= \frac{c-a}{b-a} f_{AVE}[a,c] + \frac{b-c}{b-a} f_{AVE}[c,b]$$

Section 7.1:

6. $$\star = \int (x-1)\sin(\pi x)dx \qquad u = x-1 \quad du = dx \qquad dv = \sin(\pi x)dx \quad v = -\frac{1}{\pi}\cos(\pi x)$$

$$\Rightarrow \star = \frac{1-x}{\pi}\cos(\pi x) + \frac{1}{\pi}\int \cos(\pi x)dx$$

$$= \boxed{\frac{1-x}{\pi}\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x) + C}$$

18. $\star = \int e^{-\theta} \cos(2\theta)\, d\theta \qquad u = \cos(2\theta) \qquad dv = e^{-\theta} d\theta$

$du = -2\sin(2\theta)d\theta \qquad v = -e^{-\theta}$

$\Rightarrow \star = -e^{-\theta}\cos(2\theta) - 2\int e^{-\theta}\sin(2\theta)\,d\theta$

$u = \sin(2\theta) \qquad dv = e^{-\theta}\,d\theta$

$du = 2\cos(2\theta)d\theta \qquad v = -e^{-\theta}$

$\Rightarrow \star = -e^{-\theta}\cos(2\theta) - 2\left[-e^{-\theta}\sin(2\theta) + 2\int e^{-\theta}\cos(2\theta)d\theta\right]$

$= -e^{-\theta}\cos(2\theta) + 2e^{-\theta}\sin(2\theta) - 4\star$

$\Rightarrow 5\star = e^{-\theta}\left(2\sin(2\theta) - \cos(2\theta)\right)$

$\Rightarrow \boxed{\star = \frac{1}{5e^{\theta}}\left(2\sin(2\theta) - \cos(2\theta)\right)}$

40. $\star = \int_0^{\pi} e^{\cos t}\sin 2t\, dt = \int_0^{\pi} e^{\cos t}(2\sin t\cos t)\,dt \qquad u = \cos t$

$du = -\sin t\, dt$

$\Rightarrow \star = -2\int_1^{-1} e^u \cdot u\, du = 2\int_{-1}^{1} u\cdot e^u\, du$

$= 2\left[ue^u\Big|_{-1}^{1} - \int_{-1}^{1} e^u\, du\right] = 2\left[e + \frac{1}{e} - \left[e^u\Big|_{-1}^{1}\right]\right]$

$= 2\left[\cancel{e} + \frac{1}{e} - \cancel{e} + \frac{1}{e}\right] = \boxed{\frac{4}{e}}$

48. (a). $\int \cos^n(x)dx = \int \cos(x)\cos^{n-1}(x)dx$

$= \sin(x)\cos^{n-1}(x) + (n-1)\int \sin^2(x)\cos^{n-2}(x)dx$

$= \sin(x)\cos^{n-1}(x) + (n-1)\int (1-\cos^2(x))\cos^{n-2}(x)dx$

$= \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)dx - (n-1)\int \cos^n(x)dx$

$$\Rightarrow \quad n\int \cos^n(x)\,dx = \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,dx$$

$$\Rightarrow \quad \int \cos^n(x)\,dx = \frac{1}{n}\sin(x)\cos^{n-1}(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

$\square$

(b.) $$\int \cos^2(x)\,dx = \frac{1}{2}\sin(x)\cos(x) + \underbrace{\frac{1}{2}\int \cos^0(x)\,dx}_{C}$$

$$= \frac{1}{2}\sin(x)\cos(x) + C$$

(c.) $$\int \cos^4(x)\,dx = \frac{1}{4}\sin(x)\cos^3(x) + \frac{3}{4}\int \cos^2(x)\,dx$$

(By part b)

$$= \frac{1}{4}\sin(x)\cos^3(x) + \frac{3}{8}\sin(x)\cos(x) + C$$

52. $$\int x^n e^x\,dx \qquad u = x^n \quad dv = e^x\,dx$$
$$du = nx^{n-1}\,dx \quad v = e^x$$

$$\therefore \int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$$

$\square$

64. $$V = 2\pi \int_1^3 y(\ln y)\,dy \qquad u = \ln y \quad dv = y\,dy$$
$$du = \frac{1}{y}dy \quad v = \frac{1}{2}y^2$$

$$\Rightarrow V = 2\pi\left[\frac{1}{2}y^2 \ln y \Big|_1^3 - \frac{1}{2}\int_1^3 y\,dy\right]$$

$$= \pi\left[9\ln(3) - \underbrace{\cancel{\ln(1)}}_{0}\right] - \pi\left[\frac{1}{2}y^2\Big|_1^3\right]$$

$$= 9\pi\ln(3) - \frac{\pi}{2}\left[9-1\right] = \boxed{9\pi\ln(3) - 4\pi}$$

70. $\int_0^a f(x)g''(x)\,dx = f(x)g'(x)\Big|_0^a - \int_0^a f'(x)g'(x)\,dx$

$= f(a)g'(a) - \underbrace{f(0)}_{0}g'(0) - \left[f'(x)g(x)\Big|_0^a - \int_0^a f''(x)g(x)\,dx\right]$

$= f(a)g'(a) - \left[f'(a)g(a) - f'(0)\underbrace{g(0)}_{0}\right] + \int_0^a f''(x)g(x)\,dx$

$= f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$