MATH 171-501 Homework#7

<u>Section 7.8</u>:

Determine whether each integral is divergent or convergent. Evaluate those that are convergent.

6. $\int_0^\infty (1+x)^{-1/4}\,dx = \lim_{t\to\infty} \int_0^t (1+x)^{-1/4}\,dx = \lim_{t\to\infty} \left[\frac{4}{3}(1+x)^{3/4}\right]_0^t$

$= \lim_{t\to\infty} \left[\frac{4}{3}(1+t)^{3/4} - \frac{4}{3}\right] = \infty$. Divergent

7. $\int_{-\infty}^0 \frac{1}{3-4x}\,dx = \lim_{t\to-\infty} \int_t^0 \frac{1}{3-4x}\,dx = \lim_{t\to-\infty} \left[-\frac{1}{4}\ln|3-4x|\right]_t^0$

$= \lim_{t\to-\infty} \left[-\frac{1}{4}\ln(3) + \frac{1}{4}\ln|3-4t|\right] = \infty$. Divergent

8. $\int_1^\infty (2x+1)^{-3}\,dx = \lim_{t\to\infty} \int_1^t (2x+1)^{-3}\,dx = \lim_{t\to\infty} \left[-\frac{1}{4}(2x+1)^{-2}\right]_1^t$

$= \lim_{t\to\infty} \left[-\frac{1}{4}(2t+1)^{-2} + \frac{1}{36}\right] = 0 + \frac{1}{36}$. Convergent

10. $\int_{-\infty}^0 2^r\,dr = \lim_{t\to-\infty} \int_t^0 2^r\,dr = \lim_{t\to-\infty} \left[\frac{2^r}{\ln(2)}\right]_t^0$

$= \lim_{t\to-\infty} \left[\frac{1}{\ln(2)} - \frac{2^t}{\ln(2)}\right] = \frac{1}{\ln(2)} - 0$. Convergent

16. $\int_0^\infty \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty} \int_0^t \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty} \left[-e^{\cos\theta}\right]_0^t$

$= \lim_{t\to\infty} \left[-e^{\cos t} + e\right]$ which oscillates indefinitely.

$\therefore$ Divergent.

18. $\int_2^{\infty}(v^2+2v-3)^{-1}\,dv = \lim_{t\to\infty}\int_2^t \frac{dv}{(v+3)(v-1)}$

$= \lim_{t\to\infty}\int_2^t \left(\frac{-1/4}{v+3}\right)+\left(\frac{1/4}{v-1}\right)dv$ (using partial fractions)

$= \lim_{t\to\infty}\left[-\frac{1}{4}\ln|v+3| + \frac{1}{4}\ln|v-1|\right]_2^t$

$= \lim_{t\to\infty}\left[\frac{1}{4}\ln\left(\frac{v-1}{v+3}\right)\right]_2^t = \lim_{t\to\infty}\left[\frac{1}{4}\ln\left(\frac{t-1}{t+3}\right)-\frac{1}{4}\ln\left(\frac{1}{5}\right)\right]$

$= 0-\frac{1}{4}\ln\left(\frac{1}{5}\right)=\frac{1}{4}\ln(5)$. Convergent.

30. $\int_{-1}^{2}\frac{x}{(x+1)^2}\,dx = \lim_{t\to -1^+}\int_t^2 \frac{x}{(x+1)^2}\,dx = \lim_{t\to -1^+}\int_t^2\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx$

(partial fractions)

$= \lim_{t\to -1^+}\left[\ln|x+1| + \frac{1}{x+1}\right]_t^2$

$= \lim_{t\to -1^+}\left[\ln(3)+\frac{1}{3}-\ln|1+t|-\frac{1}{1+t}\right] = -\infty$ (Justify with L'H)

Divergent.

32. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\left[\sin^{-1}(x)\right]_0^t$

$= \lim_{t\to 1^-}\left[\sin^{-1}(t)-\sin^{-1}(0)\right]=\frac{\pi}{2}$. Convergent.

36. $\int_0^4 \frac{dx}{x^2-x-2} = \int_0^4\frac{dx}{(x-2)(x+1)} = \int_0^2\frac{dx}{(x-2)(x+1)}+\int_2^4\frac{dx}{(x-2)(x+1)}$

Considering only the integral over $[0,2]$, we have: (partial fractions)

$\int_0^2\frac{dx}{(x-2)(x+1)} = \lim_{t\to 2^-}\int_0^t\left(\frac{1/3}{x-2}-\frac{1/3}{x+1}\right)dx$

$= \lim_{t\to 2^-}\left[\frac{1}{3}\ln|x-2|-\frac{1}{3}\ln|x+1|\right]_0^t = \lim_{t\to 2^-}\left[\frac{1}{3}\ln|t-2|-\frac{1}{3}\ln|t+1|\right]$

$=-\infty$. $\because$ this integral is divergent, the entire integral Diverges.