MATH 172-501 Homework 9

Section 11.1:

For problems 23, 24 and 27, determine if the sequence converges or diverges. If it converges, find the limit.

23. $a_n = \dfrac{3+5n^2}{n+n^2} = \dfrac{\frac{3}{n^2}+5}{\frac{1}{n}+1} \xrightarrow{n\to\infty} \dfrac{0+5}{0+1} = 5.$

(converges)

24. $a_n = \dfrac{3+5n^2}{1+n} = \dfrac{\frac{3}{n}+5n}{\frac{1}{n}+1} \xrightarrow{n\to\infty} \infty.$

(diverges)

27. $a_n = 3^n \cdot 7^{-n} = \left(\dfrac{3}{7}\right)^n \xrightarrow{n\to\infty} 0$ by $\boxed{9}$ in the book with $r = \frac{3}{7} < 1$

(convergent)

For problems 72-75 and 77, determine if the sequence is strictly increasing, strictly decreasing or not monotonic. Is it bounded?

72. $a_n = \cos(n) \approx \{0.54, -0.42, -0.99, -0.65 \ldots\}$, so the sequence is not monotonic.
$\because -1 \le \cos(n) \le 1 \ \forall n \in \mathbb{N}$, the sequence is bounded.

73. $a_n = \dfrac{1}{2n+3}$ is decreasing $\because a_{n+1} = \dfrac{1}{2n+5} < \dfrac{1}{2n+3} = a_n \ \forall n \ge 1.$

The sequence is bounded $\because \ 0 < a_n \le \frac{1}{5} = a_1 \ \forall n \ge 1.$

74. $a_n > a_{n+1} \iff \frac{1-n}{2+n} > \frac{1-(n+1)}{2+(n+1)} \iff -n^2-2n+3 > -n^2-2n$

$\iff 3>0$, which is true $\forall n \geq 1$, so $\{a_n\}$ is decreasing.

$\because a_1 = 0$ and $\lim_{n\to\infty} \frac{1-n}{2+n} = -1$, $\{a_n\}$ is bounded.

75. The terms of $a_n = n(-1)^n$ are alternating in sign, so $\{a_n\}$ is not monotonic.

$\because \lim_{n\to\infty} |a_n| = \infty$, the sequence is not bounded.

77. $a_n = 3-2ne^{-n}$. Let $f(x) = 3-2xe^{-x}$. Then

$f'(x) = 0-2\left[x(-e^{-x})+e^{-x}\right] = 2e^{-x}(x-1) > 0$

$\forall x > 0$, so f is increasing on $(1,\infty)$.

$\Rightarrow \{a_n\} = \{f(n)\}$ is increasing. The sequence is bounded below by $a_1 = 3-\frac{2}{e}$ and above by 3, so the sequence is bounded.