MATH 172 Homework 6

7.3 #22 ; 7.4 #11, 23, 25

$\boxed{7.3}$

22. $\int_0^1 \sqrt{x^2+1}\,dx$ Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

@ $x = 0$, $\tan^{-1}(0) = 0$

@ $x = 1$, $\tan^{-1}(1) = \pi/4$

$$\int_0^{\pi/4} \sqrt{(\tan\theta)^2+1}\ \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4} \sec\theta \cdot \sec^2\theta\, d\theta = \int_0^{\pi/4} \sec^3\theta\, d\theta$$

(See Example 7.2.8)

$$= \frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\sqrt{2}\cdot 1 + \ln(1+\sqrt{2}) - 0 - \ln(1+0)\right]$$

$$= \boxed{\frac{1}{2}\left[\sqrt{2} + \ln(1+\sqrt{2})\right]}$$

$\boxed{7.4}$

11. $\int_0^1 \frac{2}{2x^2+3x+1}$

$$\frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$2 = A(x+1) + B(2x+1)$

@ $x = -\frac{1}{2}$ $\quad 2 = \frac{1}{2}A \Rightarrow A = 4$

@ $x = -1$ $\quad 2 = -B \Rightarrow B = -2$

$$\int_0^1 \left(\frac{4}{2x+1} + \frac{-2}{x+1}\right) dx$$

$$2\ln|2x+1| - 2\ln|x+1|\Big|_0^1$$

$$2\ln(3) - 2\ln(2) - (\cancel{2\ln(1) - 2\ln(1)})$$

$$= \boxed{\ln\left(\frac{9}{4}\right)}$$

23. $\int \frac{10}{(x-1)(x^2+9)}\,dx$

$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$

$10 = A(x^2+9) + (Bx+C)(x-1)$

@ $x = 1$ $\quad 10 = 10A \Rightarrow A = 1$

$10 = x^2 + 9 + Bx^2 - Bx + Cx - C$

$1 + B = 0 \Rightarrow B = -1$

$C - B = 0 \Rightarrow C = -1 \qquad 9 - C = 10$ ✓

$\int \left(\frac{1}{x-1} + \frac{-(x+1)}{x^2+9} \right) dx$

$\ln|x-1| - \left[\int \left(\frac{x}{x^2+9} + \frac{1}{x^2+9} \right) dx \right]$

$u = x^2 + 9 \Rightarrow du = 2x\,dx$

$\boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| + \tan^{-1}(x/3) + C}$

25. $\int \frac{4x}{x^3+x^2+x+1}\,dx$

$\frac{4x}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$4x = (Ax+B)(x+1) + C(x^2+1)$

@ $x = -1$ $\quad -4 = 2C \Rightarrow C = -2$

$4x = Ax^2 + (A+B)x + B - 2x^2 - 2$

$A - 2 = 0 \Rightarrow A = 2$

$2 + B = 4 \Rightarrow B = 2 \qquad 2 - 2 = 0$ ✓

$\int \left(\frac{2(x+1)}{x^2+1} - \frac{2}{x+1} \right) dx$

$= \int \left(\frac{2x}{x^2+1} + \frac{2}{x^2+1} - \frac{2}{x+1} \right) dx$

$\boxed{\ln(x^2+1) + 2\tan^{-1}(x) - 2\ln|x+1| + C}$

Example 7.2.8

$$\int \sec^3 x\,dx = \int \sec x \cdot \sec^2 x\,dx$$

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad v = \tan x$

$$= \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan - \int \sec^3 x + \ln|\sec x + \tan x| + C$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$$