MATH 172, FALL 2017
EXAM II-VERSION A/B/C

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

SECTION NUMBER: __________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. This exam consists of 11 multiple choices questions worth 4 points each for a total of 44 points and 4 work out problems worth 14 points each for a total of 56 points.
4. In Part A (Problems 1-11), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam!*
5. In Part B (Problems 12-15), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer by boxing it.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
6. You must turn in both your exam and your scantron.
7. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: KEY.

Problems	Points Awarded	Points
1-11		44
12		14
13		14
14		14
15		14
TOTAL		100

Version A

1. C
2. A
3. B
4. B
5. C
6. C
7. C
8. A
9. A
10. B
11. D

Version B

1. B
2. D
3. C
4. B
5. D
6. D
7. C
8. D
9. A
10. B
11. B

Version C

1. C
2. A
3. B
4. B
5. C
6. C
7. D
8. A
9. C
10. B
11. D

Part A

1. Let $f\colon [2,\infty) \to \mathbb{R}$ be the function defined by $f(x) = \dfrac{1}{x^3}$. Which of the following statements is correct?

   (a) f is not integrable on $[2,\infty)$.

   (b) f is integrable on $[2,\infty)$ and $\displaystyle\int_2^\infty \frac{dx}{x^3} = \frac{1}{2}$.

   (c) f is integrable on $[2,\infty)$ and $\displaystyle\int_2^\infty \frac{dx}{x^3} = \frac{1}{8}$.

   (d) f is integrable on $[2,\infty)$ and $\displaystyle\int_2^\infty \frac{dx}{x^3} = \frac{1}{4}$.

   (e) None of the above

2. Let $f\colon (-\infty,0] \to \mathbb{R}$ be the function defined by $f(x) = \frac{1}{3-4x}$. Which of the following statements is correct?

   (a) f is not integrable on $(-\infty,0]$.

   (b) f is integrable on $(-\infty,0]$ and $\displaystyle\int_{-\infty}^0 \frac{dx}{3-4x} = \frac{1}{2}$.

   (c) f is integrable on $(-\infty,0]$ and $\displaystyle\int_{-\infty}^0 \frac{dx}{3-4x} = \frac{3}{4}$.

   (d) f is integrable on $(-\infty,0]$ and $\displaystyle\int_{-\infty}^0 \frac{dx}{3-4x} = -\frac{1}{4}$.

   (e) None of the above

3. Let $f_p\colon [1,\infty) \to \mathbb{R}$ be the function defined by $f_p(x) = \dfrac{1}{x^p}$. Which of the following statements is correct?

   (a) The function f_p is integrable on $[1,\infty)$ if $p = 1$.

   (b) The function f_p is integrable on $[1,\infty)$ if $p > 1$.

   (c) The function f_p is integrable on $[1,\infty)$ if $p < 1$.

   (d) The function f_p is integrable on $[1,\infty)$ if $-1 < p < 1$.

   (e) None of the above

4. Let $f\colon [0,1) \to \mathbb{R}$ be the function defined by $f(x) = \dfrac{1}{\sqrt{1-x}}$. Which of the following statements is correct?

(a) f is not integrable on $[0,1)$.

(b) f is integrable on $[0,1)$ and $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}} = 2$.

(c) f is integrable on $[0,1)$ and $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}} = -2$.

(d) f is integrable on $[0,1)$ and $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}} = 1$.

(e) None of the above

5. Let $(x_n)_{n=1}^{\infty}$ be a sequence and $\ell \in \mathbb{R}$. We say that $\lim\limits_{n\to\infty} x_n = \ell$ if

(a) there exists $\varepsilon > 0$ and there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $|x_n - \ell| < \varepsilon$.

(b) there exists $N \in \mathbb{N}$ such that for all $\varepsilon > 0$ and for all $n \geq N$, $|x_n - \ell| < \varepsilon$.

(c) for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $|x_n - \ell| < \varepsilon$.

(d) for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $|x_n - \ell| > \varepsilon$.

(e) None of the above

6. Let $(x_n)_{n=1}^{\infty}$ be a sequence. Which of the following statements is correct?

(a) If $(x_n)_{n=1}^{\infty}$ is bounded then $(x_n)_{n=1}^{\infty}$ is convergent.

(b) If $(x_n)_{n=1}^{\infty}$ is decreasing and bounded above then $(x_n)_{n=1}^{\infty}$ is convergent.

(c) If $(x_n)_{n=1}^{\infty}$ is increasing and bounded above then $(x_n)_{n=1}^{\infty}$ is convergent.

(d) If $(x_n)_{n=1}^{\infty}$ is increasing and bounded below then $(x_n)_{n=1}^{\infty}$ is convergent.

(e) None of the above

7. Let $(x_n)_{n=1}^{\infty}$ be the sequence defined by $x_n = \dfrac{5^n}{9^{n+1}}$ for all $n \geq 1$. Which of the following statements is correct?

(a) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent since it diverges to $+\infty$.

(b) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim\limits_{n\to\infty} x_n = \dfrac{1}{9}$.

(c) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim\limits_{n\to\infty} x_n = 0$.

(d) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent since there is no limit.

(e) None of the above

8. Let $(x_n)_{n=1}^{\infty}$ be the sequence defined by $x_n = \frac{1}{3n+2}$ for all $n \geq 1$. Which of the following statements is correct?

(a) The sequence $(x_n)_{n=1}^{\infty}$ is strictly decreasing.

(b) The sequence $(x_n)_{n=1}^{\infty}$ is increasing but not strictly increasing.

(c) The sequence $(x_n)_{n=1}^{\infty}$ is not monotone.

(d) The sequence $(x_n)_{n=1}^{\infty}$ is strictly increasing.

(e) None of the above

9. Let $(x_n)_{n=1}^{\infty}$ be the sequence defined by $x_n = \frac{3n^2+n-5}{7n^2+2n+1}$ for all $n \geq 1$. Which of the following statements is correct?

(a) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} x_n = \frac{3}{7}$.

(b) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} x_n = \frac{1}{2}$.

(c) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} x_n = 0$.

(d) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent.

(e) None of the above

10. Let $(x_n)_{n=1}^{\infty}$ be the sequence defined by $x_n = (-1)^n n^2$ for all $n \geq 1$. Which of the following statements is correct?

(a) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent since it diverges to $+\infty$.

(b) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent since there is no limit.

(c) The sequence $(x_n)_{n=1}^{\infty}$ is not convergent since it diverges to $-\infty$.

(d) The sequence $(x_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} x_n = 0$

(e) None of the above

11. Consider the series $\sum \left(\frac{1}{2}\right)^{n-1}$. Which of the following statements is correct?

(a) The series $\sum \left(\frac{1}{2}\right)^{n-1}$ is not convergent.

(b) The series $\sum \left(\frac{1}{2}\right)^{n-1}$ is convergent and $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2}$.

(c) The series $\sum \left(\frac{1}{2}\right)^{n-1}$ is convergent and $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = 1$.

(d) The series $\sum \left(\frac{1}{2}\right)^{n-1}$ is convergent and $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} = 2$.

(e) None of the above

PART B

12. (14 points)

(a) (6 points) Let $f\colon [1,\infty) \to \mathbb{R}$ be the function defined by $f(x) = \frac{1}{\sqrt{x}}$.
Show that the function f is not integrable on $[1,\infty)$?

Let $t \geq 1$. f is continuous on $[1,t]$ and thus integrable on $[1,t]$

$$\int_1^t \frac{dx}{\sqrt{x}} = \left[2\sqrt{x}\right]_1^t = 2\sqrt{t} - 2\sqrt{1}$$

$$= 2\sqrt{t} - 2 \xrightarrow[t \to +\infty]{} +\infty$$

Therefore f is not integrable on $[1,\infty)$.

(b) (4 points) State the divergence criterion of the comparison theorem for functions.

Let $f, g : [a,\infty) \longrightarrow \mathbb{R}$ be functions such that,

1) for all $x \in [a,\infty)$, $0 \leq f(x) \leq g(x)$

2) f is not integrable on $[a,+\infty)$.

Then g is not integrable on $[a,+\infty)$.

(c) (4 points) Let $g\colon [1,\infty) \to \mathbb{R}$ be the function defined by $g(x) = \dfrac{x+1}{x^{\frac{3}{2}}}$.
Is the function g integrable on $[1,\infty)$?

For all $x \geq 1$, $\dfrac{x+1}{x^{\frac{3}{2}}} \geq \dfrac{x}{x^{\frac{3}{2}}} = \dfrac{1}{\sqrt{x}} > 0$

By the divergence criterion and question a) above g is not integrable on $[1,\infty)$.

13. (14 points) Let $f\colon [0,\infty) \to \mathbb{R}$ the function defined by $f(x) = 2e^{-2x}$.

Is the function f integrable on $[0,\infty)$ (and if it is give $\int_0^\infty 2e^{-2x}dx$)?

Let $t \geq 0$, The function f is continuous on $[0,t]$ and thus f is integrable on $[0,t]$.

$$\int_0^t 2e^{-2x}\,dx = \left[-e^{-2x}\right]_0^t = -e^{-2t} - \left(-e^{-2\cdot 0}\right)$$

$$= -e^{-2t} + 1 \xrightarrow[t\to\infty]{} 1.$$

Therefore, f is integrable on $[0,\infty)$ and

$$\int_0^\infty 2e^{-2x}\,dx = 1$$

(a) (4 points) State the Squeeze Theorem for sequences.

If there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $x_n \leq y_n \leq z_n$ and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = \ell$,

then $(y_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} y_n = \ell$.

(b) (6 points) Prove the Squeeze Theorem for sequences.

Let $\varepsilon > 0$. By assumption, there exists $N_1 \in \mathbb{N}$ such that for all $n \geq N_1$, $|x_n - \ell| < \varepsilon$, and there exists $N_2 \in \mathbb{N}$ such that for all $n \geq N_2$, $|y_n - \ell| < \varepsilon$.

Let $N = \max\{N_1, N_2, n_0\}$. Then for all $n \geq N$,

$\ell - \varepsilon < x_n \leq y_n \leq z_n < \ell + \varepsilon$, and thus $|y_n - \ell| < \varepsilon$.

Therefore, ε being arbitrary, $(y_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} y_n = \ell$.

(c) (4 points) Let $(x_n)_{n=1}^{\infty}$ be the sequence defined by $x_n = \dfrac{\cos(n)^2}{n^2}$ for all $n \geq 1$.

Is the sequence $(x_n)_{n=1}^{\infty}$ convergent (and if it is give its limit)?

For all $n \geq 1$, $0 \leq x_n = \dfrac{\cos(n)^2}{n^2} \leq \dfrac{1}{n^2}$

But $\dfrac{1}{n^2} \xrightarrow[n\to\infty]{} 0$ and by the Squeeze Theorem

$(x_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} x_n = 0$.

14. (14 points) Is the series $\sum \frac{1}{n(n+1)}$ convergent (and if it is compute its sum)?

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad \text{(partial fraction expansion)}$$

$$\sum_{k=1}^{n} \frac{1}{n(n+1)} = \sum_{k=1}^{n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k+1}$$

$$= \sum_{k=1}^{n} \frac{1}{k} - \sum_{i=2}^{n+1} \frac{1}{i}$$

$$= 1 + \sum_{k=2}^{n} \frac{1}{k} - \sum_{i=2}^{n} \frac{1}{i} - \frac{1}{n+1}$$

$$= 1 - \frac{1}{n+1} \xrightarrow[n \to \infty]{} 1$$

Therefore $\sum \frac{1}{n(n+1)}$ is convergent and

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$