Test I

Instructions: Show all work in your bluebook. Cell phones, laptops, calculators that do linear algebra or calculus, and other such devices are not allowed.

1. (10 pts.) Find both the parametric equation for the plane passing through the three points \( P(0, 1, -1) \), \( Q(1, 1, 2) \), \( R(1, 2, 0) \) and the area of the triangle \( \triangle PQR \).

2. (10 pts.) Let \( \mathbf{v} = (1, -2, 3, 1) \) and \( \mathbf{u} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \). Find the projection of \( \mathbf{v} \) onto \( \mathbf{u} \) and find the distance from \( \mathbf{v} \) to the line \( x = t\mathbf{u} \).

3. Let \( A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 3 & 1 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix} \).
   
   (a) (10 pts.) For the system \( Ax = b \), form the augmented matrix \([A|b]\) and determine its reduced row echelon form.

   (b) (5 pts.) What are \( \text{rank}(A) \), \( \text{rank}([A|b]) \)? Which are the leading columns of \( A \)?

   (c) (5 pts.) Is the system consistent or inconsistent? If the system is consistent, find the parametric form of the solution.

4. (10 pts.) Hourly temperature readings from five remote stations are recorded as \( 5 \times 1 \) column vectors. Data analysis shows almost all of these vectors are linear combinations of the vectors in

\[ S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 2 \\ 1 \end{pmatrix} \right\}. \]

Determine whether \( \mathbf{T} = (1 \ 3 \ -2 \ 0 \ 2)^T \) can be represented in this way. If so, find three numbers that represent this vector. Are these numbers unique?
5. **(10 pts.)** Use row reduction either to find $C^{-1}$ or to show that it does not exist, given that the matrix $C$ is

$$C = \begin{pmatrix} 1 & 3 & -1 \\ -1 & -4 & 3 \\ 2 & 7 & -4 \end{pmatrix}.$$ 

6. Let $B = \begin{pmatrix} -1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \\ 0 & 2 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$.

   (a) **(10 pts.)** Use any method to evaluate $\det(B)$.

   (b) **(5 pts.)** Let $b = (-1 \ 0 \ 0 \ 1)^T$. Use Cramer’s rule to find the value of $x_4$ in the solution to $Bx = b$.

   (c) **(5 pts.)** What is the rank of $B$? Are the columns of $B$ LI or LD? Explain.

7. **(10 pts.)** Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $L(\vec{x}) = (3i - 4j + 6k) \times \vec{x}$. Show that $L$ is linear and find the matrix $3 \times 3$ matrix $A$ that represents $L$.

8. **(10 pts.)** Let $\mathcal{M}_{2 \times 2}$ be the set of $2 \times 2$ matrices, $A = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$. Determine whether or not $S = \{ A \in \mathcal{M}_{2 \times 2} \mid y = 2z \}$ is a subspace of $\mathcal{M}_{2 \times 2}$. 
