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On the behavior of solution to the Neumann and Mixed Boundary problem in Unbounded Domain

In the first part of the present talk we deal with the uniformly elliptic equation $(a^{i,j}(x)u_{x_i})_{x_j} = f$ in unbounded domain $\Omega \subset R^n$ and its solution $u(x)$ that satisfies the homogeneous Neumann condition. The domain Ω has the following structure: Assume that $\{r_m\}$ is an increasing sequence of positive numbers, $h_m = r_{m+1} - r_m$ and ratio h_{m+1}/h_m lies between positive constants C_1 and C_2. The intersection of Ω with spherical layers between the spheres of radius r_m and r_{m+1} with center at origin satisfies a certain inequality of isoperimetric type. In the present talk the following result will be presented: Set of the solutions of the Neumann problem splits up into three classes:

(i) the solutions for which upper limit tends to $+\infty$ and low limit tends to $-\infty$ when $|x| \to \infty$; moreover, it is shown that these limit are attained with nearly the same speed (for $C_1/C_2 = 1$ this convergence is not slower than the exponential one);

(ii) the solutions for each of which a constant C exist such that $\lim u(x) = C, and \quad u(x) - C$ changes its sing for large $|x|$; here convergence to C is rapid(for $C_1/C_2 = 1$ this convergence is not slower than the exponential one)

(iii) the solutions that do not change their sign for large $|x|$ and increase or decrease to $+\infty \quad or \quad -\infty$, respectively, with low speed (for $C_1/C_2 = 1$ with a linear speed) (one exception is possible here: a slow convergence to a constant)

In the second part (if time allows) possible generalization of the above “thrichotomy” on the class of the mixed boundary problem will be discussed.