

Airy T – bar integral

In[110]:= `bigairydelta[p_] := (Pi / 4) + (2 p^3 / 3)`

In[111]:= `smallairydelta[p_] := 3^(2 / 3) (Gamma[4 / 3] / Gamma[2 / 3]) p`

In[112]:= `bigairypade[p_] := (c p + b p^3 + (b c / a) p^4) / (1 + (c / a) p)`

In[113]:= `smallairypade[p_] := c p + b p^3`

In[114]:= `a = Pi / 4`

Out[114]= $\frac{\pi}{4}$

In[115]:= `b = 2 / 3`

Out[115]= $\frac{2}{3}$

In[116]:= `c = 3^(2 / 3) Gamma[4 / 3] / Gamma[2 / 3]`

Out[116]= $\frac{3^{2/3} \Gamma\left[\frac{4}{3}\right]}{\Gamma\left[\frac{2}{3}\right]}$

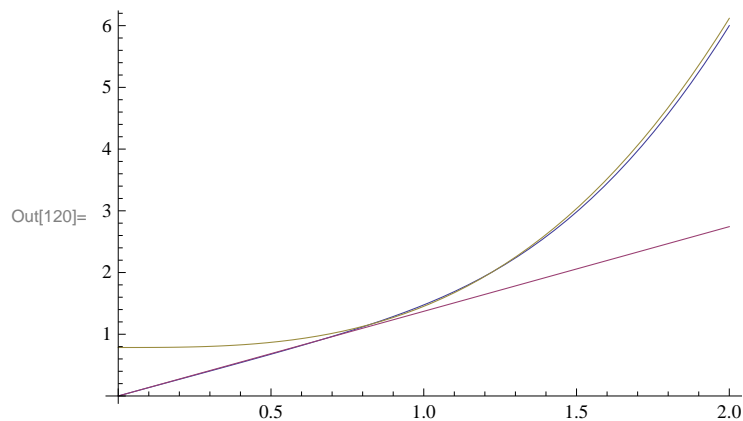
In[117]:= `airypade[p_] := h[p] bigairypade[p] + (1 - h[p]) smallairypade[p]`

In[118]:= `h[p_] := (Tanh[s (p - 0.8)] + 1) / 2`

In[119]:= `s = 1.5`

Out[119]= 1.5

In[120]:= `Plot[{airypade[p], smallairydelta[p], bigairydelta[p]}, {p, 0, 2}]`

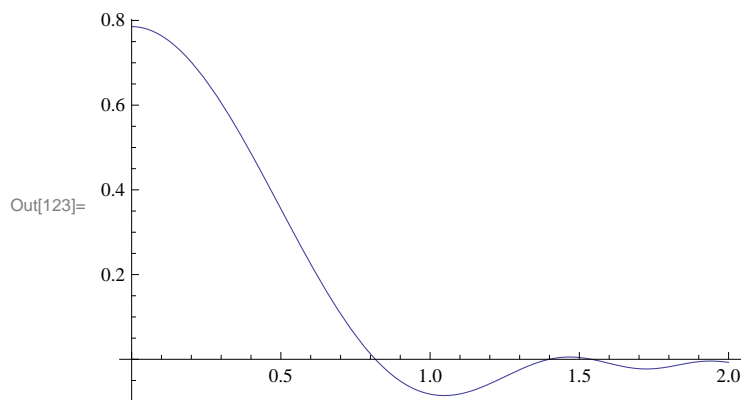


In[121]:= `uint[r_, z_] := NIntegrate[Sqrt[1 - u^2] Cos[2 z r u - 2 airypade[r u]], {u, 0, 1}]`

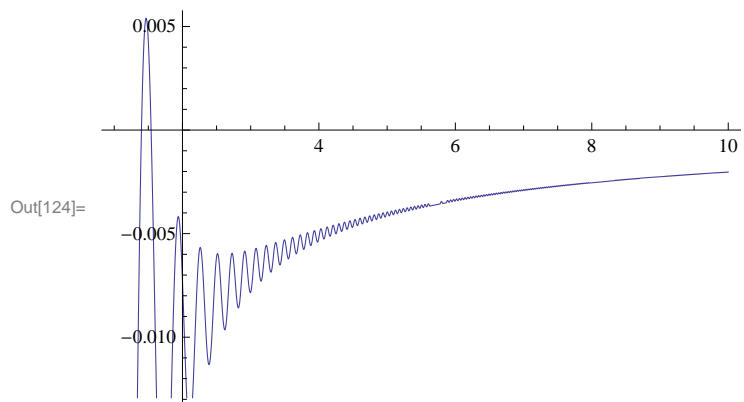
In[122]:= `uint[1, -1]`

Out[122]= -0.0822348

In[123]:= **Plot**[**uint**[**r**, -1], {**r**, 0, 2}]



In[124]:= **Plot**[**uint**[**r**, -1], {**r**, 1, 10}]

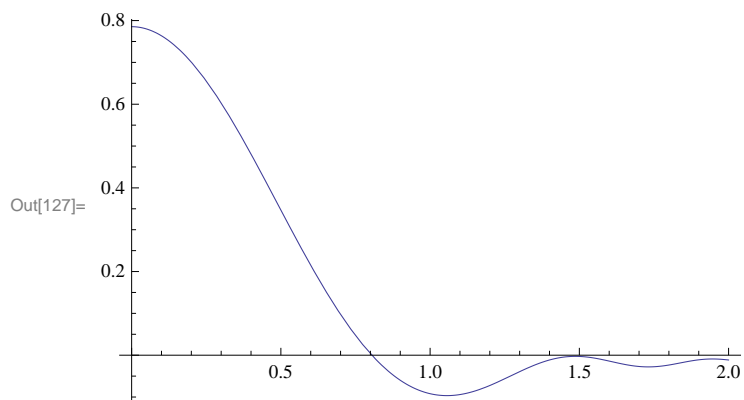


In[125]:=

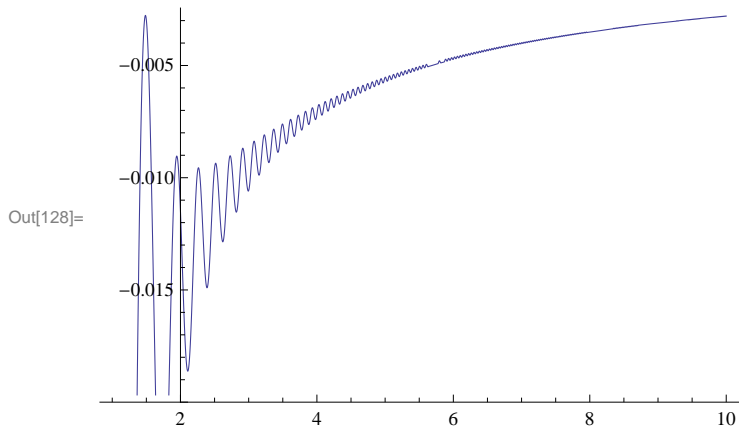
In[126]:= **s** = 2

Out[126]= 2

In[127]:= **Plot**[**uint**[**r**, -1], {**r**, 0, 2}]



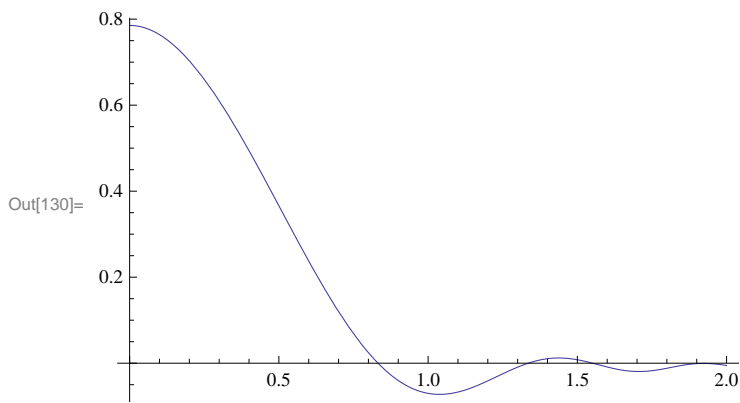
In[128]:= **Plot**[**uint**[**r**, -1], {**r**, 1, 10}]



In[129]:= **s** = 1

Out[129]= 1

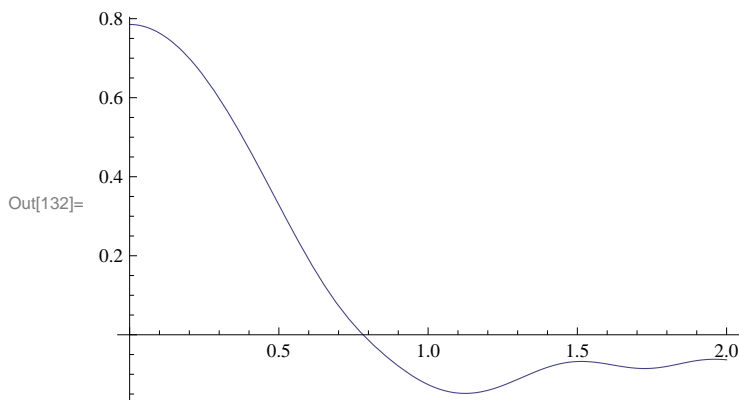
In[130]:= **Plot**[**uint**[**r**, -1], {**r**, 0, 2}]



In[131]:= **s** = 10

Out[131]= 10

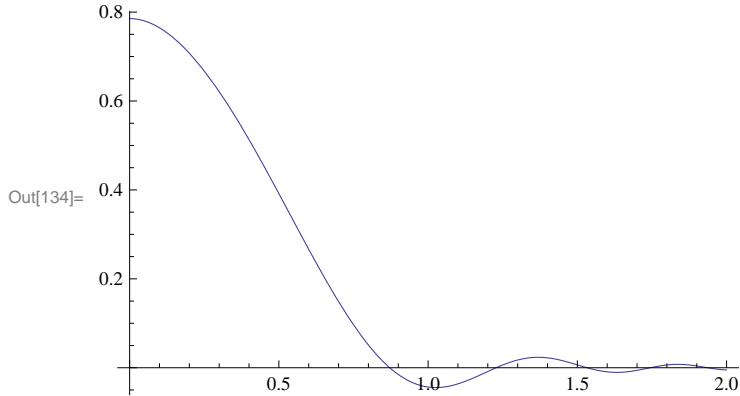
In[132]:= **Plot**[**uint**[**r**, -1], {**r**, 0, 2}]



In[133]:= **s = 0.1**

Out[133]= 0.1

In[134]:= **Plot[uint[r, -1], {r, 0, 2}]**



In[135]:= **s = 1.5**

Out[135]= 1.5

In[136]:= **tbar0[z_] := N[Pi^(-3)] NIntegrate[r uint[r, z], {r, 0, Infinity}]**

In[137]:= **tbar0[-1]**

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru + 2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle] + \text{Times}[\langle\langle 5 \rangle\rangle]\right)(1 + \text{Times}[\langle\langle 2 \rangle\rangle])\right]$$

has evaluated to non-numerical values

for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru + 2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle] + \text{Times}[\langle\langle 5 \rangle\rangle]\right)(1 + \text{Times}[\langle\langle 2 \rangle\rangle])\right]$$

has evaluated to non-numerical values

for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru + 2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle] + \text{Times}[\langle\langle 5 \rangle\rangle]\right)(1 + \text{Times}[\langle\langle 2 \rangle\rangle])\right]$$

has evaluated to non-numerical values

for all sampling points in the region with boundaries {{0, 1}}. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in r near {r} = {2616.93}. NIntegrate obtained -19157.5 and 18884.86314384178` for the integral and error estimates. >>

Out[137]= -617.858

In[138]:= **tbar0[-0.1]**

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[0.2 r u + 2 \left((\text{Times}[\llcorner 3 \gg] + \text{Times}[\llcorner 5 \gg]) (1 + \text{Times}[\llcorner 2 \gg]) + \frac{(\llcorner 1 \gg) \llcorner 1 \gg}{2 \llcorner 1 \gg} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[0.2 r u + 2 \left((\text{Times}[\llcorner 3 \gg] + \text{Times}[\llcorner 5 \gg]) (1 + \text{Times}[\llcorner 2 \gg]) + \frac{(\llcorner 1 \gg) \llcorner 1 \gg}{2 \llcorner 1 \gg} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[0.2 r u + 2 \left((\text{Times}[\llcorner 3 \gg] + \text{Times}[\llcorner 5 \gg]) (1 + \text{Times}[\llcorner 2 \gg]) + \frac{(\llcorner 1 \gg) \llcorner 1 \gg}{2 \llcorner 1 \gg} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following:
singularity, value of the integration is 0, highly
oscillatory integrand, or WorkingPrecision too small. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in r near $\{r\} = \{36.6328\}$. NIntegrate obtained 1.2128261018365734' and 1.3146999115166635' for the integral and error estimates. >>

Out[138]= 0.

In[139]:= **tbar0[-5]**

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[10 r u + 2 \left((\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]) (1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{(\llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[10 r u + 2 \left((\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]) (1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{(\llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[10 r u + 2 \left((\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]) (1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{(\llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket} \right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}\}$. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in r near $\{r\} = \{14.5189\}$. NIntegrate obtained $0.08039136405585337'$ and $0.0613056142888911'$ for the integral and error estimates. >>

Out[139]= 0.00259274

In[140]:= **riesztbar0[z_, lambda_] := N[Pi^(-3) (1 - (z / lambda))] NIntegrate[r uint[r, z], {r, 0, lambda}]**

In[141]:= **riesztbar0**[-1, 10]

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right)(1 + \text{Times}[\ll 2 \gg])\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right)(1 + \text{Times}[\ll 2 \gg])\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right)(1 + \text{Times}[\ll 2 \gg])\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following:
singularity, value of the integration is 0, highly
oscillatory integrand, or WorkingPrecision too small. >>

Out[141]= -0.00372092

In[142]:= **tbar0**[z_] := N[Pi^(-3)] NIntegrate[r uint[r, z], {r, 0.5, 3}]

In[143]:= **tbar0[-1]**

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]}{2 \llbracket 1 \rrbracket}\right)(1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{\llbracket 1 \rrbracket \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket}\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{0, 1\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]}{2 \llbracket 1 \rrbracket}\right)(1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{\llbracket 1 \rrbracket \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket}\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{0, 1\}$. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{\text{Times}[\llbracket 3 \rrbracket] + \text{Times}[\llbracket 5 \rrbracket]}{2 \llbracket 1 \rrbracket}\right)(1 + \text{Times}[\llbracket 2 \rrbracket]) + \frac{\llbracket 1 \rrbracket \llbracket 1 \rrbracket}{2 \llbracket 1 \rrbracket}\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{0, 1\}$. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

Out[143]= -0.00102573

In[144]:= **uint[0.5, -1]**

Out[144]= 0.355142

In[145]:= **uint[0, -1]**

Out[145]= 0.785398

In[146]:= **NIntegrate**[**uint**[**r**, -1], {**r**, 0, 2}]

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\right] \left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right) (1 + \text{Times}[\ll 2 \gg]) +$$

for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\right] \left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right) (1 + \text{Times}[\ll 2 \gg]) +$$

for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\ll 1 \gg) \ll 1 \gg}{2 \ll 1 \gg}\right)\right] \left(\text{Times}[\ll 3 \gg] + \text{Times}[\ll 5 \gg]\right) (1 + \text{Times}[\ll 2 \gg]) +$$

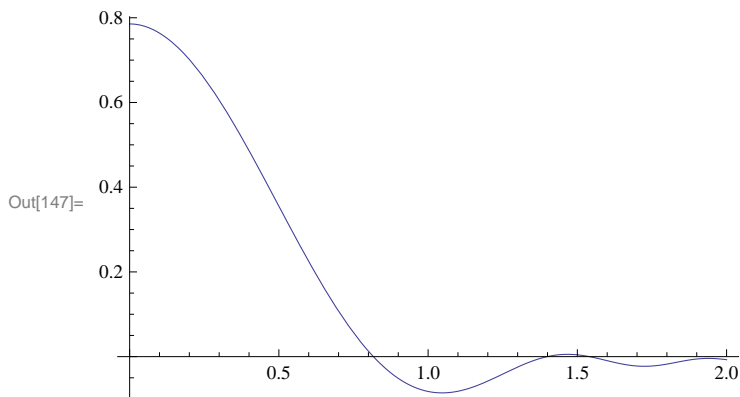
for all sampling points in the region with boundaries {{0, 1}}. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

Out[146]= 0.329923

In[147]:= **Plot**[**uint**[**r**, -1], {**r**, 0, 2}]



In[148]:= **NIntegrate**[**uint**[**r**, -1], {**r**, 0.5, 0.7}]

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

Out[148]= 0.0453826

In[149]:= **NIntegrate**[**uint**[**r**, -1], {**r**, 0.5, 0.55}]

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

NIntegrate::inumr : The integrand

$$\sqrt{1-u^2} \cos\left[2ru+2\left(\frac{(\langle\langle 1 \rangle\rangle)\langle\langle 1 \rangle\rangle}{2\langle\langle 1 \rangle\rangle}\right)\left(\text{Times}[\langle\langle 3 \rangle\rangle]+\text{Times}[\langle\langle 5 \rangle\rangle]\right)\left(1+\text{Times}[\langle\langle 2 \rangle\rangle]\right)\right]$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

General::stop :

Further output of NIntegrate::inumr will be suppressed during this calculation. >>

Out[149]= 0.0161005

In[150]:= **NIntegrate**[**x + 1**, {**x**, 0, 1}]

Out[150]= 1.5

In[151]:= **NIntegrate**[**NIntegrate**[**x + y**, {**x**, 0, 1}], {**y**, 1, 2}]

NIntegrate::inumr :

The integrand **x + y** has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 1}}. >>

Out[151]= 2.

In[152]:= **NIntegrate**[**Integrate**[**x + y**, {**x**, 0, 1}], {**y**, 1, 2}]

Out[152]= 2.

In[153]:= **uint**[**r_**, **z_**] := **Integrate**[**Sqrt**[1 - **u**²] **Cos**[2 **z r u** + 2 **airyfade**[**r u**]], {**u**, 0, 1}]

In[154]:= **tbar0**[-1]

NIntegrate::inumr :

The integrand $r \int_0^1 \sqrt{1 - u^2} \cos[2 r u - 2 (\text{Times}[\ll 2 \gg] + \text{Times}[\ll 4 \gg])] du$ has evaluated to non-numerical values for all sampling points in the region with boundaries {{0.5, 3}}. >>

Out[154]= 0.0322515 **NIntegrate**[**r uint**[**r**, -1], {**r**, 0.5, 3}]

In[155]:= **NIntegrate**[**x + y**, {**x**, 0, 1}, {**y**, 1, 2}]

Out[155]= 2.

In[156]:= **tbar**[**z_**] := **NIntegrate**[**r Sqrt**[1 - **u**²] **Cos**[2 **z r u** - 2 **airyfade**[**r u**]], {**u**, 0, 1}, {**r**, 0, **Infinity**}]

In[157]:= **tbar**[-1]

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or **WorkingPrecision** too small. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 18 recursive bisections in **r** near {**u**, **r**} = {0.00986073, 7.21074 × 10¹⁰⁷}. **NIntegrate** obtained 7.106858002330926²⁰⁹ and 4.840127101309269²¹⁰ for the integral and error estimates. >>

Out[157]= 0. × 10²⁰⁹

In[158]:= **tbar**[-0.1]

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or **WorkingPrecision** too small. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 18 recursive bisections in **r** near {**u**, **r**} = {0.00986073, 7.21074 × 10¹⁰⁷}. **NIntegrate** obtained 7.106858002330926²⁰⁹ and 4.840127101309269²¹⁰ for the integral and error estimates. >>

Out[158]= 0. × 10²⁰⁹

In[159]:= **tbar[-10]**

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following:
singularity, value of the integration is 0, highly
oscillatory integrand, or WorkingPrecision too small. >>

NIntegrate::ncvb :

NIntegrate failed to converge to prescribed accuracy after 18 recursive
bisections in r near {u, r} = {0.00986073, 7.21074 × 10¹⁰⁷}.
NIntegrate obtained 7.106858002330926²⁰⁹ and
4.840127101309269²¹⁰ for the integral and error estimates. >>

Out[159]= 0. × 10²⁰⁹

In[160]:= **s**

Out[160]= 1.5

In[161]:= **s**

Out[161]= 1.5

In[162]:= **uint[r, z]**

$$\text{Out[162]= } \int_0^1 \sqrt{1-u^2} \cos \left[2 r u z + 2 \left(\left(\frac{2 r^3 u^3}{3} + \frac{3^{2/3} r u \Gamma\left[\frac{4}{3}\right]}{\Gamma\left[\frac{2}{3}\right]} \right) \left(1 + \frac{1}{2} (-1 - \text{Tanh}[1.5 (-0.8 + r u)]) \right) \right) + \right. \\ \left. \left(\left(\frac{2 r^3 u^3}{3} + \frac{3^{2/3} r u \Gamma\left[\frac{4}{3}\right]}{\Gamma\left[\frac{2}{3}\right]} + \frac{8 r^4 u^4 \Gamma\left[\frac{4}{3}\right]}{3^{1/3} \pi \Gamma\left[\frac{2}{3}\right]} \right) (1 + \text{Tanh}[1.5 (-0.8 + r u)]) \right) \right] / \\ \left. \left(2 \left(1 + \frac{4 \cdot 3^{2/3} r u \Gamma\left[\frac{4}{3}\right]}{\pi \Gamma\left[\frac{2}{3}\right]} \right) \right) \right] du$$

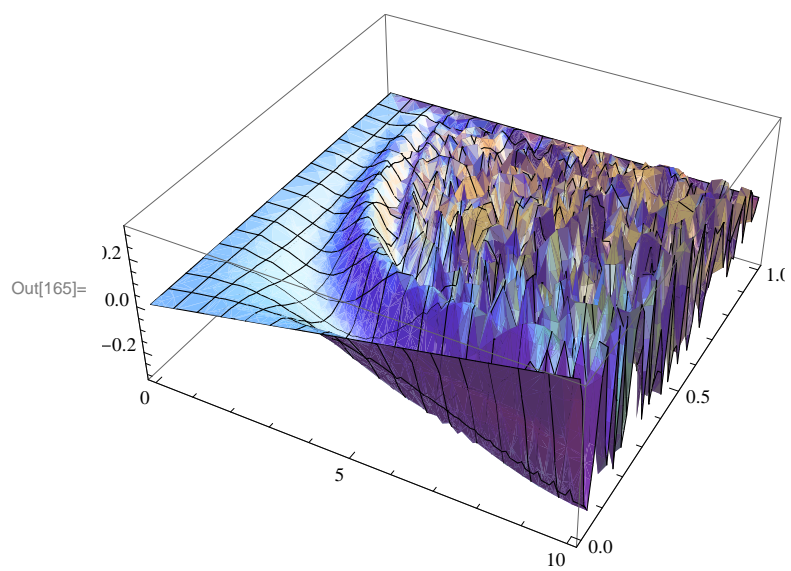
In[163]:= **? uint**

Global 'uint

`uint[r_, z_] := ∫01 √(1-u2) Cos[2 z r u + 2 airypade[r u]] du`

In[164]:= **ruint[u_, r_, z_] := Pi⁽⁻³⁾ r √(1-u²) Cos[2 z r u - 2 airypade[r u]]**

```
In[165]:= Plot3D[ruint[u, r, -1], {r, 0, 10}, {u, 0, 1}]
```



```
In[166]:= Plot3D[ruint[u, r, -1], {r, 1, 3}, {u, 0, 1}]
```

