Solutions of Einstein's equations with cylindrical symmetry

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- Relevant to describing the metric outside a cosmic string
- Perform calculations analogous to the textbook treatment of the spherically symmetric case

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- Φ, Λ, and Ψ are unknown functions of r only
- Range of ϕ runs from 0 to ϕ_* (not necessarily 2π); can be made to be 0 to 2π by rescaling ϕ

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$$\Gamma^{t}{}_{tr} = \Gamma^{t}{}_{rt} = \Phi'$$
$$\Gamma^{r}{}_{tt} = \Phi' e^{2(\Phi-\Lambda)}$$
$$\Gamma^{r}{}_{rr} = \Lambda'$$
$$\Gamma^{r}{}_{\phi\phi} = -re^{-2\Lambda}$$
$$\Gamma^{r}{}_{zz} = -\Psi' e^{2(\Psi-\Lambda)}$$
$$\Gamma^{\phi}{}_{r\phi} = \Gamma^{\phi}{}_{\phi r} = \frac{1}{r}$$
$$\Gamma^{z}{}_{rz} = \Gamma^{z}{}_{zr} = \Psi'$$

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- $R^{\alpha}{}_{\beta\mu\nu} = \Gamma^{\alpha}{}_{\beta\nu,\mu} - \Gamma^{\alpha}{}_{\beta\mu,\nu} + \Gamma^{\alpha}{}_{\sigma\mu}\Gamma^{\sigma}{}_{\beta\nu} - \Gamma^{\alpha}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\beta\mu}$

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\begin{aligned}
R^{t}{}_{\phi\phi t} &= r\Phi' e^{-2\Lambda} \\
R^{r}{}_{\phi\phi r} &= -r\Lambda' e^{-2\Lambda} \\
R^{r}{}_{zzr} &= (\Psi'' + \Psi'^2 - \Psi'\Lambda')e^{2(\Psi-\Lambda)} \\
R^{r}{}_{ttr} &= -(\Phi'' + \Phi'^2 - \Phi'\Lambda')e^{2(\Phi-\Lambda)} \\
R^{z}{}_{ttz} &= -\Psi'\Phi' e^{2(\Phi-\Lambda)} \\
R^{z}{}_{\phi\phi z} &= r\Psi' e^{-2\Lambda}
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$$R_{tt} = (\Phi'' + \Phi'^2 - \Phi'\Lambda' + \frac{1}{r}\Phi' + \Psi'\Phi')e^{2(\Phi-\Lambda)}$$

$$R_{rr} = -\Phi'' - \Phi'^2 + \Phi'\Lambda' + \frac{1}{r}\Lambda' - \Psi'' - \Psi'^2 + \Lambda'\Psi'$$

$$R_{\phi\phi} = r(\Lambda' - \Phi' - \Psi')e^{-2\Lambda}$$

$$R_{zz} = -(\Psi'' + \Psi'^2 - \Psi'\Lambda' + \Psi'\Phi' + \frac{1}{r}\Psi')e^{2(\Psi-\Lambda)}$$

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$$\Lambda' = \Omega' = \Phi' + \Psi'$$

$$\Phi'' + \frac{1}{r}\Phi' = 0$$

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$$\Phi'\Psi' + \frac{1}{r}\Phi' + \frac{1}{r}\Psi' = 0$$

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- After rescaling t, z, and r to absorb constants (and with $a := a_1$, $b := b_1$):
  $ds^2 = -r^{2a} dt^2 + r^{2(a+b)} dr^2 + K^2 r^2 d\phi^2 + r^{2b} dz^2$ - leads to two conventions for ϕ

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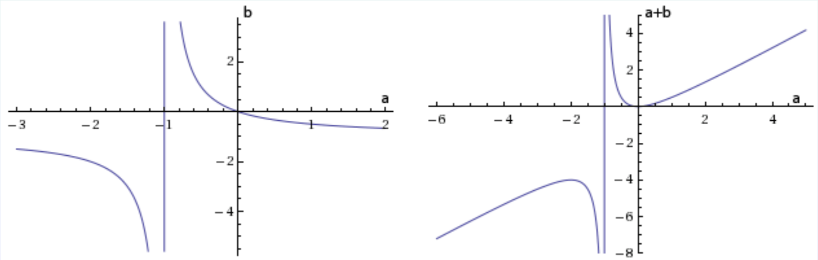

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- Also reduces to: $ds^2 = -d\bar{t}^2 + d\bar{r}^2 + \bar{r}^2 d\bar{\phi}^2 + d\bar{z}^2$

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- Thus in all three cases, $r = 0$ in our metric corresponds to $\bar{r} = \infty$ in the alternate metric whenever $a < -1 \Leftrightarrow b < -1 \Leftrightarrow a + b < -1$

Future Work

- Look at $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$

Future Work

- Look at $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$
- Try connecting exterior solution to string of finite radius

Thank You

- Questions?