The simple star graph consists of $N$ edges meeting at a common vertex, each edge of length 1, with a real valued potential $q_i \in L^\infty(0,1)$ defined on the $i$th edge, with edge parameterized by $[0,1]$ in such a way that $x = 1$ corresponds to the common vertex where the edges meet. The direct eigenvalue problem consists in finding $\lambda \in \mathbb{C}$ and $\psi(x) = \{\psi_1(x), \psi_2(x), \psi_3(x)\}$ on $[0,1]$, such that $\psi''_i(x) + q_i(x)\psi_i(x) = \lambda \psi_i$ where $\psi_i(0) = 0$ and we have Kirchoff matching conditions at the common vertex $x = 1$. These $\{\lambda := \lambda_k\}_{k=1}^\infty$ are the graph spectra.

There are several inverse problems which seek to recover $q_i(x)$. All assume we are given the graph spectra, but this is insufficient and additional information is required. We shall look at the case where we also know the Dirichlet spectrum of each node.

A good physical way to view this problem is as $N$ unit strings each of unknown density $\rho_i$ joined at a single node and clamped at the other end. We are given the vibrational frequencies of the system together with the frequencies of each edge when the node is also clamped. Can this be used to recover the $\rho_i$?