High Frequency Vibrations of a Generalized Cylindrical Nonlinear Elastic Tube

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Abstract

Results are presented on the analysis of high frequency, small amplitude vibrations of a generalized cylindrical nonlinear elastic tube. The problem derives from modeling the use of an ultrasound, catheter to interrogate in vivo atherosclerotic plaques in large arteries. The artery is modeled as a thick walled tube described parametrically in cylindrical coordinates (applied to a reference configuration) by two surfaces $s_I(R, \theta, Z) < s_O(R, \Theta, Z)$ for $0 \leq Theta < 2\pi$, $0 \leq Z \leq L$ representing the inner and outer arterial wall surfaces, respectively. The arterial wall is modeled as a residually stressed, nonlinear elastic body. The tube is first subjected to a static, deformation resulting from a uniform pressure applied to the inner wall surface (modeling the effect of uniform fluid pressure). Both fixed grip and third kind boundary conditions are considered for the outer wall. Next a small amplitude, high frequency, time harmonic vibration is superimposed on the inner wall's static pressure distribution. The problem is analyzed via linearization about the static large deformation of the residually stressed tube. The goal of the analysis is the determination of the vibrational frequency spectrum's dependence upon tube geometry, material properties and static deformation from the unloaded (but residually stressed) configuration. Subsequently, the inverse problem is investigated by which one attempts to determine the tube's geometry and stiffness from "measured" natural vibrational frequencies.