Math 417, Homework 5

1. Let $u(x, y)$ be the steady state temperature on a square plate which is heated on one side and cooled on the other three sides. The temperature $u(x, y)$ satisfies

$$u_{xx} + u_{yy} = 0, \quad (x, y) \in (0, 1) \times (0, 1).$$

The boundary conditions are

$$u(x, 1) = 1, \quad u(x, 0) = u(0, y) = u(1, y) = 0.$$

Compute the solution using the 5-point Laplacian with mesh size $h = 1/4,\ 1/8$. For each value of h, solve the linear system using Jacobi, Gauss-Seidel and optimal SOR iteration. To compute ω_*, use the fact that $\rho(B_J) = \cos(\pi h)$. The starting guess should be zero at the interior points and should agree with the boundary conditions at the boundary points. Terminate the iteration when the maximum residual is less than 10^{-2}. To avoid an infinite loop, set the maximum number of iterations to 100. For each method (J, GS, SOR) make a table which displays the following information:

column 1: h (mesh size)

column 2: k (number of iterations needed)

column 3: $\|r_k\|_\infty$ (norm of residual at final step)

column 4: $\|r_k\|_\infty / \|r_{k-1}\|_\infty$ (ratio of successive residual norms at final step)

Summarize the results. Are the results consistent with the theorems discussed in class?

Plot the solution.

Hints:

a. In solving the linear system do not form the entire matrix. Take advantage of the fact that the typical equation involves only 5 unknowns.

b. Organize your code the following way:

for icase =1:2

set h, n

initialize u

iterate on u until convergence criterion is satisfied

plot the solution

end

c. Jacobi requires 2 vectors: u_{new}, u_{old}. Gauss-Seidel and SOR require just 1 vector.