Math 417, Homework 7

1. Let $f(x) = 1/x$. Take $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$,

   (a) Find the Lagrange form, the Newton form and the standard form of the interpolating polynomial p_3. Check your answer by veryfying that p_3 correctly interpolates f at the given points.

   (b) Use the theorem proven in class to find an upper bound for the maximum error

$$\|f - p_3\|_\infty = \max_{1 \le x \le 4} |f(x) - p_3(x)|.$$

2. Let x_0, x_1, x_2 be 3 distinct points. Find the standard form of the plynomial

$$p(x) = \ell_0(x) + \ell_1(x) + \ell_2(x).$$

   Solve this problem two ways: first by direct computation, second by applying the theorem which says that there is a unique polynomial of degree $\le n$ which interpolates a given function at $n+1$ distinct points.

3. For $f(x) = e^x$ the following data is given:

$$f(0) = 1, \quad f'(0) = 1, \quad f(1) = 2.7182818, \quad f'(1) = 2.7182818.$$

   Estimate $f(0.5)$ using linear and Hermite interpolation. Which estimate is more accurate?

4.. Let $x_0, \ldots, x_n$ be distinct points. In Hermite interpolation, the function $\tilde{h}_j(x)$ was defined to be the polynomial of degree $2n+1$ satisfying

$$\tilde{h}_j(x_i) = 0, \quad \tilde{h}'_j(x_i) = 0, \ i \ne j, \quad \tilde{h}'_j(x_j) = 1, \quad i, j = 0, \ldots, n.$$

   (a) Verify that $\tilde{h}_j(x) = (x - x_j)\ell_j(x)^2$, where $\ell_j(x)$ is the Lagrange polynomial.

   (b) Let $n = 1$, $x_0 = 0$, $x_1 = 1$. Use the expression given in (a) to find $\tilde{h}_1(x)$.

   (c) Use the divided difference table to find $\tilde{h}_1(x)$. Plot $\tilde{h}_1(x)$ (by hand).

5. Write a program to perform polynomial interpolation at the uniform points and the Chebyshev points on the interval $[-1, 1]$ for the functions

$$f_1(x) = |x|, \quad f_2(x) = sign(x).$$

   ($sign(x) = 1$ if $x > 0$, $sign(x) = 0$ if $x = 0$, and $sign(x) = -1$ if $x < 0$) Investigate the convergence of p_n to f by running the program for different values of n. In the write up include plots of f and p_n for $n = 8$, 16, 32 for both sets of points. Answer the following questions.

   Does p_n converge pointwise to f on $[-1, 1]$?

   Does p_n converge uniformly to f on $[-1, 1]$?