An axiomatic system example. Assume that a club of two or more students is organized into committees in such a way that each of the following conditions are satisfied.

a) Every committee is a set of one or more students.

b) For each pair of students, there is exactly one committee on which both serve.

c) No single committee is composed of all students in the club.

d) Given any committee and any student not on that committee, there exists exactly one committee on which that student serves which has no students of the first committee in its membership.

Prove each of the following, justifying each step of your proof to one of the four conditions or one of the statements previously proven.

i Every student serves on at least two committees.

ii Every committee has at least two members.

iii There are at least four students in the club.

iv There are at least six committees in the club.

Add an additional condition.

e) No committee has more than two members.

Prove:

v There are exactly four members in the club.

vi There are exactly six committees in the club.

***Postulate* 7** d is a function from $S \times S \to \Re$

***Postulate* 8** For every P, Q, $d(P,Q) \geq 0$. We will write $d(P,Q)$ as just PQ for ease of use. Don't confuse this with $\overleftrightarrow{PQ}$ which is a line.

***Postulate* 9** $d(P,Q) = 0$ if and only if (iff) $P = Q$.

***Postulate* 10** $d(P,Q) = d(Q,P)$ for every P and Q in S.

***Postulate* 11**

Ruler Postulate [3.3.d-4] Every line has a coordinate system.

Definition 11

Betweenness Let A, B, C be three collinear points. If $AB + BC = AC$ then B is between A and C. We write $A - B - C$.

***Theorem* 5** If $A - B - C$ then $C - B - A$

Lemma 1 Given a line L with a coordinate system and three points A,B, C with y between x and z then $A - B - C$.

***Theorem* 6** Of any three points on a line exactly one is between the other two.

If we write $A - B - C - D$ (with w, x, y, z being coordinates respectively) then either $w < x < y < z$ or $w > x > y > z$. And we extend this notation as necessary for more points.

***Theorem* 7** Any four points of a line can be named such that $A - B - C - D$.

***Theorem* 8** If A and B are any two points, then

1. there is a point C such that $A - B - C$, and
2. there is a point D such that $A - D - B$.

Homework 3

1. Show that if $A - B - C$ and $B - C - D$ then $A - B - D$ and $A - C - D$.
2. Show that if $A - B - C$ and $A - D - C$ then $A - B - D - C$ or $A - D - B - C$ or $B = D$.
3. Given four beads of different colours. In how many different ways is it possible to arrange them in a trough, in order from left to right. How many ways is it possible to arrange them on a rigid symmetrical rod? How many different ways to arange them on a circular loop?

4. Which postulate is illustrated by each of the following, if A,B, C are different points, k and m are two lines, and P is a plane?
   (a) If $A \in P, C \in P, A \in k$ and $C \in k$, then $k \in P$
   (b) If $B \in k, C \in k$ and $k \neq m$ then B and C cannot both be on m.
   (c) If $A \in k, B \in k, A \in m$, and $B \in m$ then $k = m$.
   (d) There is a point C such that $C \notin P$.
   (e) If $A \in k, B \in k$, and $C \notin k$ then there is a plane P such that $A \in P$, $B \in P$ and $C \in P$.