Definition 7

Logical System consists of undefined terms, definitions, assumptions and theorems.

The undefined terms for geometry are **set**, **point**, **line**, **plane**.

Definition 8

One-to-one correspondence is if two sets have the same number of elements. If two sets have a one-to-one correspondence (written 1-1) then they are said to be equivalent.

Definition 9

$\aleph$ is the symbol for the natural numbers. The ones that start with $\{1, 2, 3, \cdots\}$ and continue on.

Definition 10

Finite A set is finite if it is equivalent to a set of the form $\{1, 2, 3, 4 \cdots, m\}$ for some $m \in \aleph$. Otherwise it is infinite.

The basic structure of a geometry contains the set S, the set of points, the set L, the set of lines, and the set P, the set of planes. With just this structure there is no guarantee that we will get the familiar geometry of Euclid. The only thing we will let ourselves know about these sets is in the following postulates. These are called incidence postulates because they define how the various parts of the geometry interact.

***Postulate* 1** All lines and planes are sets of points.

***Postulate* 2** Given any two different points there is exactly one line containing them. If P and Q are two points then the line is denoted $\overleftrightarrow{PQ}$.

***Postulate* 3** Given any three different non-collinear points, there is exactly one plane containing them. If P, Q and R are points then the plane containing them is $\overleftrightarrow{PQR}$.

***Postulate* 4** If two points lie in a plane then the line containing them lies in the plane.

***Postulate* 5** If two planes intersect, then their intersection is a line.

***Postulate* 6** Every line contains at least two points. S contains at least three non-collinear points. Every plane contains at least three non-collinear points. And S contains at least four non-coplanar points.

Note that all of the statements reading two points, two planes, etc. it may occasionally mean that the two things could actually be the same. Though all of the above postulates imply two different items.

***Theorem* 1** Two different lines intersect in at most one point.

***Theorem* 2** If a line intersects a plane not containing it, then the intersection is a single point.

***Theorem* 3** Given a line and a point not on the line, there is exactly one plane containing both of them.

***Theorem* 4** If two different lines intersect, then their union lies in exactly one plane.

Homework 2

1. Consider the space with exactly four points {a,b,c,d}, in the positions of a tetrahedron (triangular pyramid). Verify the incidence postulates.
2. Let p_1, p_2, p_3, p_4, p_5, be five points, no three of which are collinear. How many lines are there? If no four of the five points are coplanar, how many planes are there?
3. Expand the previous two problems to n points. It might help to start with four and then add a point and attempt to find a pattern.
4. Show that under the incedence postulates, the space S cannot be a line.
5. Show that there is at least one plane.
6. Show that there are at least two planes.