

**Definition 1**

**Negation** If  $p$  is a statement, the statement  $\sim p$  is the negation of  $p$ .

**Example 1** Form the negation of the following statements.

- a The moon is rising.
- b  $\angle ABC$  is a remote interior angle.
- c Point  $C$  is between points  $A$  and  $B$ .
- d  $m\angle 3 = 25^\circ$

Solution

- a The moon is not rising.
- b  $\angle ABC$  is not a remote interior angle.
- c Point  $C$  is not between points  $A$  and  $B$ .
- d  $m\angle 3 \neq 25^\circ$

**Definition 2**

**Conjunction** If  $p$  and  $q$  are statements the compound statement ' $p$  and  $q$ ' is the conjunction.

**Example 2** Indicate whether the following statements and their conjunctions are true or false.

- a Dogs have five feet. Cats have six feet.
- b Every line contains at least one point.  
Every plane contains all the points of space.
- c Every line contains at least two points.  
Every plane contains at least three points.

Solutions

- a Both of the statements are false, so the conjunction is false.
- b The statement about the plane is false so the conjunction is false.
- c Both statements are true so the conjunction is true.

**Definition 3**

**Disjunction** If  $p$  and  $q$  are statements then ' $p$  or  $q$ ' is the disjunction of  $p$  and  $q$ .

Use the previous pairs of statements and determine if the disjunctions are true or false.  
Solutions

- a Both of the statements are false so the disjunction is false.
- b One of the statements is true so the disjunction is true.
- c Both statements are true so the disjunction is true.

Theorems are often stated in if-then form. I.E. If the sky is blue then it is not raining. Or in general 'if  $p$  then  $q$ '. Where  $p$  is called the hypothesis, and  $q$  is called the conclusion. Here are some examples of if-then statements. Determine what the hypothesis and conclusion are.

**Example 3**

- a If it is cloudy, then it will rain.
- b If I eat too much, I will get sick.
- c I will be very upset if I fail geometry.
- d All crazy people like porcupines.

**Definition 4**

**Converse** If you interchange the hypothesis and the conclusion of a statement then you have formed a converse. Example of a converse of the statement in the previous paragraph If it is not raining then the sky is blue.

### Definition 5

**Inverse** If you negate the hypothesis and the conclusion you have formed the inverse. If the sky is not blue then it is raining, is an example of an inverse.

### Definition 6

**Contrapositive** The converse of the inverse is called the contrapositive (or the inverse of the converse). If it is raining then the sky is not blue.

**Example 4** Find the converse, inverse and contrapositive of 'It is raining if the sky is cloudy'.

**Solution** First put the sentence in standard if-then form, 'If the sky is cloudy then it is raining'.

**converse** If it is raining then the sky is cloudy.

**inverse** If the sky is not cloudy then it is not raining.

**contrapositive** If it is not raining then it is not cloudy.

### Homework 1

1. Write the hypothesis and conclusion to each of the following. Also write the converse, inverse and contrapositive.
  - (a) If I earn enough money then I will buy a house.
  - (b) If you get fat then you ate too much.
  - (c) You will be able to study better if your SO (significant other) is not with you.
  - (d) A theorem always has a hypothesis and a conclusion.
2. Decide what is given and what is to be proved for each of the following statements.
  - (a) The supplements of congruent angles are congruent.
  - (b) If two lines intersect, the bisectors of a pair of adjacent angles formed are perpendicular.
  - (c) The bisectors of the three angles of a triangle intersect in the same point.