Probability Theory, Math 411, Section 503, Fall 2017 -
Homework 2

From the textbook solve the problems 14,15,17,19 at the end of the Chapter 1. And also the problems below:

Problem 1. Two players take turns removing a ball from a jar that initially contains m white and n black balls. The first player to remove a white ball wins. Find the probability that the starting player wins.

Problem 2. Alice and Bob have $2n+1$ coins, each coin with probability of heads equal to $1/2$. Bob tosses $n+1$ coins, while Alice tosses the remaining n coins. Assuming independent coin tosses, show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

Problem 3. Suppose that we draw 2 cards out of a deck of 52. Let $A =$"the first card is an ace" and $B =$"the second card is a spade". Are A and B independent?

Problem 4. A family has 3 children, each of whom is a boy or a girl with probability $1/2$. Let $A =$"there is at most 1 girl" and $B =$"the family has children of both sexes". (a) Are A and B independent? (b) Are A and B independent if the family has 4 children?

Problem 5. Let A and B be two independent events with $P(A) = 0.4$ and $P(A \cup B) = 0.64$. What is $P(B)$?

Problem 6. On a lottery ticket you choose and circle six numbers out of the numbers 1, 2, 3, ... 44, 45. The next day at the lottery six numbers are chosen at random (with all combinations having the same probability) out of the numbers 1, 2, 3, ... 44, 45. You win the prize if on your lottery ticket you have guessed exactly three out of this six numbers. What is the probability you win the prize.

Problem 7. You have an 8×8 chessboard and a token at the low left corner square. You want to move it to the top right corner square. In the questions below you need to compute in how many ways you can do that given the restrictions, that is how many such token trajectories exists.

a) In how many ways can you move the token if, at each move you're only allowed to move the token one square to the right or one square up?

b) In how many ways can you move the token if in addition you're also allowed to move the token diagonally (in one move you can also move it to the adjacent "right up" square).

Problem 8. Assume that $0 \le m \le n$. Give a combinatorial proof that

$$\binom{n}{m} = \sum_{k=m}^{n} \binom{k-1}{m-1}.$$

(Hint: how many m-element subsets of $\{1, 2, \ldots, n\}$ are there which have k as the largest element?) Plug in $m = 2$ here, what familiar formula do you get?

Note that the right hand side $\sum_{k=m}^{n} \binom{k-1}{m-1}$ is just another way to write

$$\binom{m-1}{m-1} + \binom{m}{m-1} + \binom{m+1}{m-1} + \cdots + \binom{n-1}{m-1}.$$