Math 411 - Fall 2014 - 2nd Exam.

1. (25 pts) Determine which of the following statements are true or false:
   - a) Let $X$ be a discrete random variable that takes values $1, 2, 3$. The distribution function $F_X$ is not continuous at the points $1, 2, 3$.
   - b) Let $X, Y$ be two random variables. Then $\text{var}(X + Y) = \text{var}(X) + \text{var}(Y)$.
   - c) Let $U$ be uniformly distributed on $[0, 2]$. Let $a, b \in [\frac{1}{2}, \frac{3}{2}]$ and $n \geq 2$. Then
     \[ P\left( a - \frac{1}{n} < U < a + \frac{1}{n} \right) = P\left( b - \frac{1}{n} < U < b + \frac{1}{n} \right). \]
   - d) Let $X$ be a hypergeometric random variable with parameters $n, N, m$. ($n$ is the size of the “sample”, $N$ is the total population and $m$ is the size of the “target” population). If $n$ is very small with respect to $N, m$, then $X$ can be approximated by a Binomial r.v. $Y$ with parameters $n, \frac{m}{N}$.
   - e) If $X$ is normal with parameters $\mu, \sigma^2$ and $a, b$ are real numbers, then $Y = aX + b$ is also normal with $E[Y] = a\mu + b$.

2. (20 pts) Let $X$ be a discrete random variable taking values $\{1, 2, 3, 4\}$ with equal probabilities. Write the probability mass function and the distribution function of $X$.
   a) Compute the probabilities: $P\{1 \leq X \leq 3\}, P\{1 < X \leq 3\}$ and $P\{1 < X < 2\}$.
   b) Compute the expected value of $Y := X^2$.

3. (20 pts) Let $X$ be a random variable with density
   \[ f_X(t) := a \begin{cases} 
   t & 0 < t \leq 1 \\
   2 - t & 1 < t \leq 2 \\
   0 & \text{otherwise}
   \end{cases} \]
   where $a$ is a real number. Compute a) $P(X \leq \frac{1}{2})$ and b) $E[X]$.

4. (15 pts) Let $U$ be uniformly distributed on $[0, 1]$. Find the density of the random variable $U^2$.

5. (20 pts) Suppose that we roll two dice 12 times and let $D$ be the number of times a double 6 appears. a) Compute the probabilities $P(D = 0)$ and $P(D = 2)$. b) Use Poisson to provide an approximation of the probabilities $P(D = 0)$ and $P(D = 2)$.

6. (20 pts) a) Suppose that a man’s height has a normal distribution with mean $\mu = 69$ (inches) and standard deviation $\sigma = 3$. What is the probability that a randomly chosen man is more than 72 inches tall?
   b) Suppose that we flip a (fair) coin 100 times. Use the (DeMoivre) central limit theorem to compute the probability to get at least 56 heads. (Use continuity correction).

Write 100 out of the 120 available points