

## Mini-Courses

### **Radoslaw Adamczak**

*Random matrices with independent log-concave rows/columns.*

I will present some recent results concerning geometric properties of random matrices with independent rows/columns distributed according to a log-concave probability measure. In particular I will discuss the following topics: 1. Estimates on the operator norm of such matrices 2. Solution to the Kannan-Lovasz-Simonovits problem (i.e. providing a condition on a matrix with independent rows to be an almost isometry) 3. Restricted isometry property for matrices with independent columns and neighbourliness of random polytopes with vertices sampled from isotropic convex bodies 4. Estimates on the smallest singular value of a square matrix (if time permits). While focusing on the probabilistic part of the above problems I will try to indicate connections with geometry of convex bodies and log-concave measures as well as some geometric motivations and applications (e.g. sampling and computing the volume of high dimensional convex bodies, compressed sensing). The talks will be based on joint work with O. Guedon, A. Litvak, N. Tomczak-Jaegermann and A. Pajor.

### **Sasha Sodin**

*Some applications of the moment method in random matrix theory*

We shall discuss some applications of the moment method to eigenvalue statistics of random matrices (hopefully, from a unified point of view). (I) Introduction, the basic combinatorial technique, Wigner's law (with Wigner's original proof), and perhaps the results of Kesten and McKay (on the spectrum of  $d$ -regular graphs). (II) The distribution of extreme eigenvalues: Soshnikov's theorem. We shall state the theorem, explain the basic idea of Soshnikov's original proof, and then – in more detail – a proof from a joint work with Ohad Feldheim. (III) Sparse random matrices and periodic band matrices. We shall try to explain the motivation to study these classes of matrices and discuss some results from the physical literature. Then we shall state and prove some rigorous results on the eigenvalues close to the edge of the spectrum and the corresponding eigenvectors. No preliminary knowledge is presumed.

## **Roman Vershynin**

### *Sums of independent random matrices*

This series of talks will describe Ashwede-Winter's method, which produces deviation inequalities for sums of independent random matrices. This simple method is parallel to the classical exponential concentration inequalities (e.g. Chernoff's) for sums of independent random variables. The crucial tool to work with random matrices rather than random variables in Golden-Thomson trace inequality. Ashwede-Winter method implies in particular Rudelson's sampling theorem for random vectors in the isotropic position.

## **Mahler Conjecture and Reverse Santalo**

## **Greg Kuperberg**

### *From the Mahler conjecture to Gauss linking integrals*

The Mahler volume of a centrally symmetric convex body  $K$  in  $n$  dimensions is the product of the volume of  $K$  and the polar body  $K^\circ$ . Mahler conjectured that the Mahler volume is maximized by ellipsoids and minimized by cubes. The upper bound is the Blaschke-Santaló inequality. Bourgain and Milman showed that the lower bound, known as the Mahler conjecture, is true up to an exponential factor.

I will describe a proof of the Bourgain-Milman theorem that establishes the Mahler conjecture up to an exponential factor of  $(\pi/4)^n$ . The proof minimizes a different volume at the opposite end of the space of convex bodies, i.e., at ellipsoids. If time permits, I will also discuss another table-turning conjecture of this type that would imply the isotropic constant conjecture with a good constant.

## **Dmitry Ryabogin**

### *Nazarov's proof of Bourgain-Milman Theorem.*

Recently Fedor Nazarov gave a new proof of the Bourgain-Milman Theorem: there exists a constant  $c > 0$  independent of the dimension such that  $(\text{vol}_n(K)\text{vol}_n(K^*))^{1/n} \geq c(\text{vol}_n(B_\infty^n)\text{vol}_n(B_1^n))^{1/n}$ . The main idea of the proof is to use the Paley-Wiener-Schwarz Theorem. The constant  $c$  in this proof is  $\pi^3/4^3$ , which is worse than the best known  $c = \pi/4$ , obtained by G. Kuperberg. Nevertheless, this new approach represents an independent interest. I will present Nazarov's proof.

## Talks

### Tim Austin

*Some recent results on non-conventional ergodic averages*

Since Furstenberg gave a proof of Szemerédi's Theorem in additive combinatorics by showing its equivalence to a multiple recurrence result in ergodic theory, the nonconventional ergodic averages that are the subjects of this result and its later generalizations have attracted considerable interest. In this talk we will discuss some recent progress in their analysis, showing how an extension of an initially-given system of commuting probability-preserving transformations can be used in a proof of the norm convergence of some such averages.

### Julio Bernues

*On the isotropy constant of projection of polytopes.*

The isotropy constant of any  $d$ -dimensional polytope with  $n$  vertices is bounded by  $C\sqrt{\frac{n}{d}}$  where  $C > 0$  is a numerical constant. This is joint work with D. Alonso, J. Bastero and P. Wolff.

### Nikos Dafnis

*Small ball probability estimates,  $\psi_2$  behavior and the hyperplane conjecture*

We introduce a method which leads to upper bounds for the isotropic constant. We prove that a positive answer to the hyperplane conjecture is equivalent to some very strong small probability estimates for the Euclidean norm on isotropic convex bodies. As a consequence of our method, we obtain an alternative proof of the result of J. Bourgain that every  $\psi_2$ -body has bounded isotropic constant, with a slightly better estimate: If  $K$  is a convex body in  $\mathbb{R}^n$  such that  $\|\langle \cdot, \theta \rangle\|_q \leq \beta \|\langle \cdot, \theta \rangle\|_2$  for every  $\theta \in S^{n-1}$  and every  $q \geq 2$ , then  $L_K \leq C\beta\sqrt{\log \beta}$ , where  $C > 0$  is an absolute constant. This is joint work with G. Paouris.

### Daniel John Fresen

*A Glivenko-Cantelli theorem in metric spaces*

We discuss Glivenko-Cantelli theorems for empirical and Poisson processes in metric spaces; not in the usual setting of uniform convergence, but with respect to Monge-Kantorovich distances. We use these ideas to study the empirical ellipsoid of inertia of randomly chosen points in Euclidean space.

**Alexander Koldobsky**

*Positive definite functions and multidimensional versions of random variables.*

**Greg Kuperberg**

*How much entropy is there in quantum non-locality?*

Quantum probability is a natural generalization of standard probability. Any one random variable is exactly as it was before, but random variables no longer have to commute. Remarkably, quantum probability is empirically true, in the same sense that classical probability is true in science. We simply do not usually see it, because most of our affairs lie within a commutative realm of the non-commutative reality.

Quantum probability is also not a superficial generalization: it cannot exist within classical probability. One rigorous version of this conclusion is that quantum probability violates Bell-type inequalities. However, traditional Bell-type protocols, which are also called non-locality demonstrations, are statistically inefficient. They consume many quantum bits of data for each bit of persuasion. I will describe some new protocols that are more efficient than these traditional protocols. If time permits, I will also discuss related open questions that have some flavor of asymptotic geometry.

**Mark Meckes**

*Concentration of polynomial functions of random matrices*

In the spirit of results of Guionnet and Zeitouni and of free probability theory, we prove concentration inequalities for noncommutative polynomials of large independent random matrices. This is joint work with S. Szarek.

**Peter Pivovarov**

*On the volume of random polytopes generated by points in an isotropic convex body*

Let  $K \subset \mathbb{R}^n$  be an isotropic convex body and let  $X_1, \dots, X_N$  be independent random vectors distributed uniformly in  $K$ . I will discuss lower bounds for the volume of the random polytope  $\text{conv}\{X_1, \dots, X_N\}$  and the zonotope  $\sum_{i=1}^N [-X_i, X_i]$ . I will focus on the dependence of estimates on the isotropic constant  $LK$  and the relation to the Slicing Problem.

**Dmitry Ryabogin**

*On the Mahler conjecture, local minimality of the unit cube*

This is a joint work with F. Nazarov, F. Petrov and A. Zvavitch. We prove that the unit cube  $B_\infty^n$  is a strict local minimizer for the Mahler volume product  $vol_n(K)vol_n(K^*)$  in the class of  $n$ -dimensional origin symmetric convex bodies  $K$  endowed with the Banach-Mazur distance.

**Jakub Wojtaszczyk**

*Properties of permutation-invariant convex bodies*