1. Let $A, B$ two events with Probability $P(A) = 0.2$ and $P(B) = 0.9$. Decide if the following statements are true or false:

- a) $A$ and $B$ have common elements.
- b) It is certain that either $A$ or $B$ will happen.
- c) $A$ is a subset of $B$.

Explain your answer.

2. Two fair dice are rolled. What is the probability of the event $E := \text{the second die lands on a higher value than does the first}$? What is the probability that the event $F := \text{sum is 9}$? Compute the probability of $E \cap F$ and of $E \cup F$.

3. 10 people are in a room. What is the probability that no two of them celebrate their birthday in the same month?

4. Five people, $(A, B, C, D, E)$ are arranged randomly in linear order. What is the probability that there is exactly one person between $A$ and $B$? What is the probability that there are at least 3 people between $A$ and $B$?

5. An urn contains 4 red, 5 white and 6 black balls and 4 balls are randomly chosen, without replacement. Finds the probability that at least one ball of each color has been chosen.

Due to Monday September 22nd .

**Hints-Solutions**

1. a) True. $P(AB) = P(A) + P(B) - P(A) = 1.1 - P(A \cup B) \geq 0.1$. b) c) False. Consider the experiment that you roll a die with 10 faces. Consider the events $A_1 = \text{less than 3}', B_1 = \text{less than 10}'$. $A_2 = \text{more than 8}', and B_1 again.$

2. $P(E) = 15/36, P(F) = 4/36. P(E \cup F) = 17/36, P(E \cap F) = 2/36.$

3. (We assume that all monthly outcomes are equal.) We work as example 5i. Probability $= \frac{12!}{212^{11}}$.

4. a) $\frac{3 \times 3!}{3!}$, b) $\frac{2 \times 3!}{3!}$. 
5. Let \( R= \) “no red selected”, \( W= \) “no white selected”, \( B= \) “no black selected”. Let \( A \) the event that at least one ball of each color selected. Then \( P(A) = 1 - P(R \cup W \cup B) \). Also

\[
P(R \cup W \cup B) = P(R) + P(W) + P(B) - P(RW) - P(RB) - P(WB) + P(RWB)
\]

We have that \( P(RWB) = 0 \), \( P(R) = \left( \frac{9}{15} \right) \) and \( P(RW) = \left( \frac{3}{15} \right) \). We compute the rest terms in a similar way.