Math 411 - Fall 2014 - 2nd Homework.

1. Let $A, B$ two events with Probability $P(A) = 0.2$ and $P(B) = 0.9$. Decide if the following statements are true or false:
   - a) $A$ and $B$ have common elements.
   - b) It is certain that either $A$ or $B$ will happen.
   - c) $A$ is a subset of $B$.

   Explain your answer.

2. Two fair dice are rolled. What is the probability of the event $E := \text{“the second die lands on a higher value than does the first”}$? What is the probability that the event $F := \text{“sum is 9”}$? Compute the probability of $E \cap F$ and of $E \cup F$.

3. 10 people are in a room. What is the probability that that no two of them celebrate their birthday in the same month?

4. Five people, (A, B, C, D, E) are arranged randomly in linear order. What is the probability that there is exactly one person between A and B? What is the probability that there are at least 3 people between A and B?

5. An urn contains 4 red, 5 white and 6 black balls and 4 balls are randomly chosen, without replacement. Finds the probability that at least one ball of each color has been chosen.

Due to Monday September 22nd.

Hints-Solutions

1. a) True. $P(AB) = P(A) + P(B) - P(A) = 1.1 - P(A \cup B) \geq 0.1$. b)c) False. Consider the experiment that you roll a die with 10 faces. Consider the events $A_1 = \text{“less than 3”}, B_1 = \text{“less than 10”}. A_2 = \text{“more than 8”}, \text{and B}_1 \text{ again}$.

2. $P(E) = 15/36, P(F) = 4/36. P(E \cup F) = 17/36, P(E \cap F) = 2/36.$

3. (We assume that all monthly outcomes are equal.) We work as example 5i. Probability $= \frac{12!}{2!(12)^{10}}$.

4. a) $\frac{3 \times 3!}{5!}$, b) $\frac{2 \times 3!}{5!}$.
5. Let $R$=“no red selected”, $W$=“no white selected”, $B$=“no black selected”. Let $A$ the event that at least one ball of each color selected. Then $P(A) = 1 - P(R \cup W \cup B)$. Also

$$P(R \cup W \cup B) = P(R) + P(W) + P(B) - P(RW) - P(RB) - P(WB) + P(RWB)$$

We have that $P(RWB) = 0$, $P(R) = \frac{\binom{9}{4}}{\binom{15}{4}}$ and $P(RW) = \frac{\binom{5}{4}}{\binom{15}{4}}$. We compute the rest terms in a similar way.