Math 411 - Fall 2014 - 1st Homework.

1. Consider the set \( A = \{ A, B, C, D, E, F, G \} \). How many subsets of \( A \) of size 3 we have? How many of these subsets contain at least one of the letters \( A \) or \( B \)?

2. How many 5-digit numbers are there for which
   
   - a) the digit 7 appears at most 2 times.
   - b) each digit appears at most 1 time.
   - c) no two consecutive digits are equal.

   *Remark:* 00000 is considered a 5-digit number in our case.

3. In how many way a teacher can devide 20 exercises into 3 tests if the first test must have 6 problems and the other two must have 7 problems?

4. - a) How many distinct nonnegative integer valued solutions of \( x_1 + x_2 + x_3 = 4 \) are possible?
   - b) Compute how many partial derivatives of order 4 an analytic function of 3 variables has.

5. How many different linear arrangement are there of the letters \( A, B, C, D, E \) for which
   
   - a) \( A \) is last in line?
   - b) \( A \) is not last in line?
   - c) \( A \) is before \( D \)?
   - d) \( A, B \) and \( C \) are next to each other?

Due to Friday September 12.

*Hints-solutions*

1. a) \( \binom{7}{3} \).
2. a) \( 9^3 + \binom{5}{1}9^4 + \binom{5}{2}9^3 \)
   - b) \( 10 \times 9 \times 8 \times 7 \times 6 \).
   - c) \( 10 \times 9 \times 9 \times 9 \times 9 \).
3. \( \binom{20}{6,7,7} \) (e.g. see Examples 5a, 5b)
4. a) (See Proposition 6.1) \( \binom{6}{2} \) (b) same as a).
5. a) 4!, b) 5! - 4! c) \( \frac{1}{2} \!5! \), d) 3! \times 3!. 

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