Math 411 - Spring 2015 - 7th Homework.

1. Two fair dice are rolled. Find the joint probability mass function of \((X, Y)\) when \(X\) is the smallest and \(Y\) is the largest value obtained on the dice.

2. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter \(\lambda = 1\). What is
   1. \(P(X \geq 4)\)?
   2. \(P(X \geq 8|X \geq 4)\)?

3. Let \(X\) be uniformly distributed on \((0, 1)\). Find
   1. The distribution function of \(X\)
   2. The distribution function of \(Y = e^X\).
   3. The density of \(Y\).

4. The lung cancer hazard rate \(\lambda(t)\) of a \(t\)-year-old male smoker is
   \[
   \lambda(t) = 0.027 + 0.00025(t - 40)^2, \ t \geq 40.
   \]
   Assuming that a 40-year-old male smoker survives all other hazards, what is the probability that he survives to age 50?

5. The joint probability density function of \(X, Y\) is given by
   \[
   f(x, y) = e^{-(x+y)}, \ 0 \leq x, y \leq \infty.
   \]
   Find
   1. \(P(X < Y)\)
   2. \(f_X(t)\).
   3. \(E[X]\).

Due to Monday April 13.

Hints-Solutions
1) e.g. \(p(2, 3) = 2/36, \ p(5, 5) = 1/36\).
2) a) \(P(X \geq 4) = \frac{1}{e^4}\). b. Same as a. (Memoryless property)
3) a. See page 195. b. \(F_Y(t) = P(e^X \leq t) = P(X \leq \log(t)) = F_X(t)\). c. Take derivative with respect to \(t\).
4) \(P(X \geq 50|X \geq 40) = \frac{P(X \geq 50)}{P(X \geq 40)}\). Moreover
   \[
   P(X \geq t) = \exp\{- \int_0^t \lambda(s)ds\}.
   \]
5) \(P(X < Y) = \int_0^\infty \int_0^y e^{-(x+y)}dxdy = \cdots = \frac{1}{2}\).