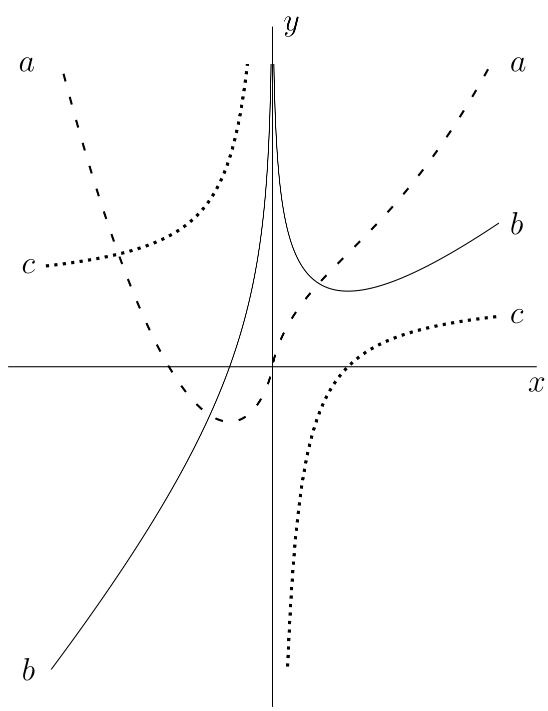
Calculus

Instructions Please write your solutions on your own paper. Explain your reasoning in complete sentences to maximize credit.

1. On what interval(s) is the function $x^2 e^x$ decreasing?

2. Suppose $f(x)$ is the function $\sin(x) + \arcsin(x)$, which is a continuous function whose domain is the closed interval $[-1, 1]$. At which point x in the domain does this function attain its absolute maximum value?

3. In the figure below, one graph represents $f(x)$, another graph represents the derivative $f'(x)$, and a third graph represents the indefinite integral (antiderivative) $\int f(x)\,dx$. Which graph is which?



4. The TI-89 calculator says that $\lim_{x\to 0} \dfrac{\arctan(2x)}{\sin(3x)} = \dfrac{2}{3}$. Prove that the calculator is correct.

5. The *error function* $\operatorname{erf}(x)$ is defined to be $\dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\,dt$. Show that the graph of $\operatorname{erf}(x)$ is concave down when $x > 0$.

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6. Sketch the graph of a function f that satisfies the following properties:

   - the domain of f is all real numbers except for ± 1;
   - the derivative $f'(x) < 0$ everywhere on its domain;
   - $\lim_{x \to 1^+} f(x) = +\infty$ and $\lim_{x \to 1^-} f(x) = -\infty$;
   - $\lim_{x \to -1^+} f(x) = +\infty$ and $\lim_{x \to -1^-} f(x) = -\infty$;
   - $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = 0$;
   - the point $(0, 0)$ is an inflection point of the graph of f, and there is no other inflection point.

7. A farmer has 1,000 linear feet of fence with which to bound a rectangular field. If one side of the field will be formed by a river, and the fence is used for the other three sides, what is the largest possible area of the field?

8. If $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \sqrt{\ln\left(1 + \frac{i}{n}\right)} = \int_1^2 f(x)\,dx$, what is the function $f(x)$?

9. State **three** of the following theorems:

   (a) l'Hospital's rule for indeterminate forms of type 0/0

   (b) the extreme value theorem

   (c) Fermat's theorem

   (d) the mean-value theorem

   (e) the first derivative test for local extrema

   (f) the first derivative test for absolute extrema

   (g) the second derivative test for local extrema

   (h) the fundamental theorem of calculus

10. **Optional problem for extra credit**
    State the remaining five theorems from problem 9.