1. Let $L$ be the linear operator on $\mathbb{R}^3$ defined by the property that

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_2 - x_3 \\ x_3 - x_1 \end{pmatrix}.$$  

Find a basis for the kernel of $L$. 


2. Suppose $L : P_2 \rightarrow P_3$ is the linear transformation defined by the property that $L(p(x)) = xp(x)$ for every polynomial $p(x)$. (Recall that $P_n$ denotes the space of polynomials of degree less than $n$.) Determine the matrix representation of $L$ with respect to the ordered basis $[1, x]$ in $P_2$ and the ordered basis $[1, 1 + x, 1 + x + x^2]$ in $P_3$. 