3.1 - Introduction to Limits

Notation: $\lim_{x \to a} f(x) = L$. This is read

I. Analyzing Limits Graphically

Recall:

From a Graph:

from the homework: $f(x) = \begin{cases} x + 3, & x < -2 \\ \sqrt{x + 2}, & x > -2 \end{cases}$

Example: $f(x) = \dfrac{|x|}{x}$

Example: $f(x) = \dfrac{x^2 - 4}{x - 2}$

graphically:

from tables:

algebraically:

II. An Algebraic Approach

Properties of Limits

Let $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Then

1. $\lim_{x \to a} [f(x)]^r = \left[\lim_{x \to a} f(x)\right]^r = L^r$

2. $\lim_{x \to a} c\,(x) = c \lim_{x \to a} f(x) = c\,L$

3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$

4. $\lim_{x \to a} [f(x)\, g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L \cdot M$

5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$, $M \neq 0$

6. $\lim_{x \to a} const = const$

Examples:

a)

b)

c)

d)

(e)

(f)

III. Limits and the Difference Quotient

3.1 # 1 – 57 (odd), 65 - 68