2.5 - EXPONENTIAL FUNCTIONS

$f(x) = b^x$, $b > 0$, $b \neq 1$, b is called the base and x is the exponent.

Graph:

Characteristics:

1. Domain:
2. Range:
3.
4.
5.

All Laws of Exponents still hold:

1. $b^x \cdot b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $\left(b^x\right)^y = b^{x \cdot y}$
4. $(ab)^x = a^x b^x$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Examples:

1. $f(x) = 2^x$, $f(-1) =$
2. $\frac{8^{5/3}}{8^{-1/3}}$
3. Solve: $2^{2x-1} = 16$

Base e:

APPLICATIONS:

Growth and Decay

$y = c\,e^{k\,t}$ $\quad k > 0 \Rightarrow$ *growth* and $k < 0 \Rightarrow$ *decay*

Exponential Growth - Populations grow exponentially. The national debt grows exponentially.

Example: Bacteria grows at a rate of 1.386. Initially, there are 25 bacteria in the population. How many bacteria will there be in 3.5 hours?

(b) If we began with 55 bacteria, how many bacteria would there be in 45 minutes?

Exponential Decay - radioactive decay and carbon 14 dating

Example: Carbon 14 decays at a rate of 0.000124. If 500 mg of carbon 14 is present in a sample of tissue now, how much will be present in 45,000 years?

COMPOUND INTEREST - An application of the exponential function to business.

Simple Interest - interest computed only on the original principal P

$$I = P\,r\,t$$

P - principal (amount of original investment)

r - interest rate

t - number of years invested

Let A be the accumulated amount. Then $A = P + I$

$= P + P\,r\,t$

$= P\,(1 + r\,t)$ simple interest formula

Earning interest on interest is called **compound interest** where

$$A = P\left(1 + \frac{r}{m}\right)^{m\,t}$$

and m = number of conversion periods per year.

Example: If $1,000 is invested at 10% monthly, how much will be in the account in 10 years?

Calculator:

Present Value:

P - principal, also called present value

A - accumulated amount, also called future value

- simply solve the previous formula for P

Continuous Compound Interest:

$y = P\, e^{r\, t}$

How long will it take money to double at 2.5% interest%?

2.5 HW # 1 – 21 (every other odd), 29 - 37 (odd), 43 – 49 (odd)