2.6 - LOGARITHMS

I. Inverse Functions - The inverse of a function f , denoted $\left(f^{-1}\right)$, reverses the effect of f.

Not every function has an inverse. Only functions that are **one-to-one** have inverses, i.e. passes the horizontal line test.

The range of f becomes the domain of f^{-1} and domain of f becomes the range of f^{-1}.

II. LOGARITHMIC FUCNTION

Graph of $f(x) = \log_b x$, $b > 0,\ b \neq 1$

Properties:

Continuous on $(0, \infty)$

Passes through (1, 0)

Domain: $(0, \infty)$

Range: $\Re$

b > 1 - increasing

0 < b < 1 - decreasing

III. Conversions

$$\log_a x = c \quad or \quad a^c = x$$

So, **LOGARITHMS ARE EXPONENTS!!** (defined for x > 0; recall graph)

A. 10 is a special base (called the common log) and is written $\log x = c \ \ or \ \ 10^c = x$

This says that the logarithm to base 10 of x is the power of 10 you need to get x. Examples of base 10: Richter scale, pH

B. The most frequently used base is base e or the **natural logarithm**.

$$\log_e x = \ln x = c \quad or \quad e^c = x$$

(**ln** x is the power of e needed to get x)

III. PROPERTIES OF LOGARITHMS

1. $\log_b(xy) = \log_b x + \log_b y$

2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

3. $\log_b x^p = p\log_b x$

4. $\log_b b^x = x$

5. $b^{\log_b x} = x$

6. $\log_b 1 = 0$ *because* $b^0 = 1$
   $\log_b b = 1$ *because* $b^1 = b$

IV. Change of Base Formula: $\log_a x = \frac{\ln x}{\ln a}$ and $e^{x\ln b} = b^x$

Function Hierarchy:

e^x	grows the fastest
x^p	(p +)
$\ln x$	grows slowest

Examples:

2.6 HW # 1 – 49 (odd), 61 – 66, 79-84
Ch. 2 review # 27-33 (odd), 51, 75, 77, 85, 87, 88