3.5 - INTRODUCTION TO THE DERIVIATIVE

I. The **Instantaneous Rate of Change** AT $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \quad \text{if the limit exists .}$$

II. **Slope of the Tangent Line** (also called the slope of the graph)

A GEOMETRIC INTERPRETATION

Recall from geometry that a tangent line to a circle is a line that passes through one and only one point on the circle. But for functions in general, this is not a satisfactory definition.

To define a tangent line for f at a point P:

1. A point P is given on f

2. Pick a point Q on f

3. Draw a line through PQ
(this is the secant line)

4. Let $Q \to P$

5. $\lim_{Q \to P} \begin{pmatrix} slopes \;\; of \;\; the \\ \sec ant \;\; lines \end{pmatrix} = m_{\tan}$

(m = slope of the tangent line at P)

III. **<u>The Derivative Function</u>**

Use point-slope form of a line to write the equation of the tangent line as follows:

If P has coordinates $(x, f(x))$, then Q has coordinates $(x+h,\ f(x+h))$ since Q is some distance h from P and

$$m_{\overline{PQ}} = \frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$$

As the distance from Q to $P \to 0$ (in other words, $h \to 0$),

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = m_{\tan} \quad \text{(the slope of the tangent line to } f \text{ at P)}$$

(also called the slope of the graph of f at P, the instantaneous rate of change, velocity and the DERIVATIVE).

SO,

$$f'(x) = \text{rate of change of } f \text{ at } x = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}, \quad \text{if the limit exists,}$$

(Alternate notations: $f'(x) \approx f' \approx y' \approx \dfrac{dy}{dx} \approx D_x$).

The notation $\dfrac{dy}{dx}$ reminds us that the derivative is a <u>rate of change</u>.

The derivative of a function f' is a new <u>function</u> whose domain is a subset of the domain of f.