

### 3.2 - Limits and Continuity

One-sided limits help define **continuity**:

A function  $f$  is continuous at a number  $x = a$  if

(1)  $f(x)$  is defined

(2)  $\lim_{x \rightarrow a} f(x)$  exists

(3)  $\lim_{x \rightarrow a} f(x) = f(x)$

If not, the function  $f$  is discontinuous at  $x = a$ .

Further,  $f$  is continuous on an interval  $I$  if it is continuous at every number in  $I$ .

Example:  $f(x) = \begin{cases} x + 1, & x \geq 0 \\ x - 1, & x < 0 \end{cases}$

Example:  $f(x) = \begin{cases} \sqrt{x}, & x > 0 \\ -x, & x \leq 0 \end{cases}$

Example:  $h(x) = \begin{cases} x + 2, & x \neq 2 \\ 1, & x = 2 \end{cases}$

Graph  $h(x)$ . Is  $h(x)$  continuous at  $x = 2$ ?