

## 2.3 - LOGARITHMS

### I. LOGARITHMIC FUNCTION

Graph of  $f(x) = \log_b x, b > 0, b \neq 1$

Properties: Continuous on  $(0, \infty)$

Passes through  $(1, 0)$

Domain:  $(0, \infty)$

Range:  $\mathfrak{R}$

$b > 1$  - increasing

$0 < b < 1$  - decreasing

### II. Conversions

$$\log_a x = c \text{ or } a^c = x$$

So, LOGARITHMS ARE EXPONENTS!!

(defined for  $x > 0$ ; recall graph)

**A.** 10 is a special base (called the common log) and is written  $\log x = c$  or  $10^c = x$

This says that the logarithm to base 10 of  $x$  is the power of 10 you need to get  $x$ .

Examples of base 10: Richter scale, pH

**B.** The most frequently used base is base  $e$  or the **natural logarithm**.

$$\log_e x = \ln x = c \text{ or } e^c = x \quad (\ln x \text{ is the power of } e \text{ needed to get } x)$$

### III. PROPERTIES OF LOGARITHMS

1.  $\log_b(xy) = \log_b x + \log_b y$
2.  $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3.  $\log_b x^p = p \log_b x$
4.  $\log_b b^x = x$
5.  $b^{\log_b x} = x$
6.  $\log_b 1 = 0$  because  $b^0 = 1$   
 $\log_b b = 1$  because  $b^1 = b$

**IV. Change of Base Formula:**  $\log_a x = \frac{\ln x}{\ln a}$  and  $e^{x \ln b} = b^x$

Function Hierarchy:

- $e^x$  grows the fastest
- $x^p$  (p + )
- $\ln x$  grows slowest

Examples: