

A few pop tests. Separated by rules.

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1. (6 points) Verify that the function  $\phi(x) = c_1e^{-x} + c_2e^{2x}$  is a solution of the equation

$$y'' - y' - 2y = 0$$

for any choice of the constants  $c_1$  and  $c_2$ .

2. (4 points) Determine  $c_1$  and  $c_2$  to satisfy the initial conditions  $y(0) = 2$ ,  $y'(0) = 1$ .
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1. (6 points) Find the general solution to

$$\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = x \cos x, \quad x > 0.$$

2. (4 points) Your solution will have a constant. Sketch the solution for two different values of the constant (say, 1 and  $-1$ ).
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1. (6 points) Solve the initial value problem

$$\frac{dP}{dt} = P(1 - \ln P), \quad P(0) = 1.$$

2. (4 points) Sketch the solution for  $0 \leq t \leq 6$ .
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1. (6 points) I will draw a mechanical “circuit” on the whiteboard. Set up a system of second order differential equations for the system.

2. (4 points) Convert the system into a system of four first order equations.

3. (extra 4 points) Write the system in 2 in matrix form.
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1. (4 points) Find the Laplace transform of the differential equation

$$y'' + y = \sin x.$$

2. (4 points) Substitute the initial conditions  $y(0) = 1$ ,  $y'(0) = 0$  in the transformed equation.

3. (4 points) Solve this equation for  $Y(s)$  (the Laplace transform of  $y$ ) and find the inverse transform to get  $y(t)$ .