

## The Normal Distribution

When we graph a probability distribution we draw a histogram. Each rectangle in the histogram has a width of 1 and a height that is the probability that  $X$  occurs. So the area, height $\times$ width, is the probability.

With the distributions we have studied  $X$  was an integer - how many heads were rolled, how many green marbles in the sample, how many hearts in your hand of cards. Here we will study random variables that are CONTINUOUS.

Continuous random variables can be things like time, weight, length, volume. Many of these quantities follow a normal (bell-shaped) curve. We will find the probability of measuring a certain value by finding the area.

We will find the area under the normal curve by using the calculator. Start with the STANDARD NORMAL CURVE. This one has a mean of 0 ( $\mu = 0$ ) and a standard deviation of 1 ( $\sigma = 1$ ). The variable associated with the standard normal curve is  $Z$ .

Example - find the following probabilities:

$$P(-3 < Z < -2)$$

$$P(Z > -1)$$

$$P(Z < 1.5)$$

When the distribution has a mean  $\mu$  and a standard deviation  $\sigma$  we use the symbol  $X$  for the random normal variable. We can still find the area under the curve. If you have a TI-83, you simply put in the value for  $\mu$  and  $\sigma$  where prompted and the range of the  $X$  values. If you have a TI-82, you must convert to a  $Z$ -score to find the area. The formula is

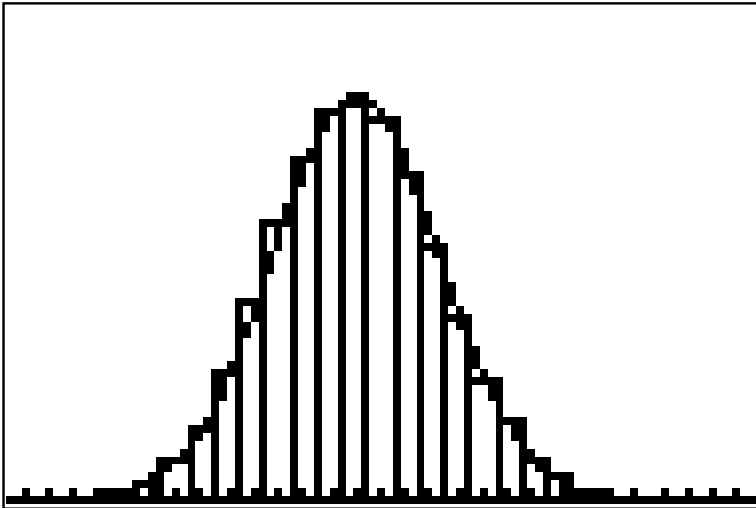
$$Z = \frac{x - \mu}{\sigma}$$

Example - IQ scores approximately follow the normal distribution. Given that the mean IQ score is 100 with a standard deviation of 15, find the following:

- what is the probability of being gifted (IQ>130)?
- what is the probability of being normal (85<IQ<115)?
- In a population of 10 million people, estimate the number of geniuses (IQ>175).

Example - consider 50 children exposed to the chicken pox. The probability that a child will catch chicken pox is 30%. Shown is the histogram for a binomial experiment with  $n = 50$  and  $p = .3$ . Drawn with it is the normal curve with

$$\mu = 50 \cdot .3 = 15 \text{ and } \sigma = \sqrt{50 \cdot .3 \cdot .7} \approx 3.2$$



Use the normal curve to APPROXIMATE the binomial distribution.

What is the probability that more than 20 children catch the chicken pox?

Look at our picture. To find the EXACT answer we would add up the probability that 21+22+...+50 children caught it.

However, we want to find the APPROXIMATE answer and so we will find the area under the normal curve that includes 21 or more children. The rectangle that represents 21 children starts at 20.5. So we will find the area from  $X = 20.5$  to  $X = 50.5$ .