WEEK 7 REVIEW

- Multiplication Principle (6.3)
- Combinations and Permutations (6.4)
- Experiments, Sample Spaces and Events (7.1)
- Definition of Probability (7.2)

WEEK 8 - 7.3, 7.4 and Test Review

THE MULTIPLICATION PRINCIPLE

At a pasta diner there is a choice of 4 different pastas and 3 different sauces. How many dinners can be made? Use a TREE DIAGRAM to organize the choices and outcomes

We find 12 different dinners and $12 = 4 \cdot 3$. What if there were 5 different meats to choose from? We need the MULTIPLICATION PRINCIPLE:

Suppose a task T_1 can be completed n_1 ways, a task T_2 can be completed n_2 ways, $\ldots$ and a task T_r can be completed n_r ways. The number outcomes from making these r choices is the product

$$n_1 \cdot n_2 \cdot \ldots n_r$$

So there would be $4 \cdot 3 \cdot 5 = 60$ possible dinners with the additonal choice of 5 meats.

Example - How many different computer addresses are possible if the first three spots have letters and the last four spots have digits?

We have 26 ways to complete the task of choosing a letter for each of the letter places and 10 ways to complete the task of choosing a digit for each of the four digit places. In all

$$26 \cdot\ 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175,760,000$$

Example - How many different 4-digit access codes can be made if the first digit cannot be a 0 or a 1 and no repeats are allowed?

The first spot has 8 choices available. Now one digit is removed from our possible digit list since we cannot have repeats, but now the 0 and 1 are available and so the next spot has 9 choices, then 8 then 7,

$8 \cdot 9 \cdot 8 \cdot 7 = 4032$

Example - How many different 2 scoop ice cream cones are possible if there are 31 flavors to choose from?

31 choices for the first scoop and 31 choices for the second scoop = $31 \cdot 31 = 961$ different two scoop cones.

PERMUTATIONS

A common application of the multiplication principle is to choose elements from a finite set and arrange them in a certain way.

How many ways can 8 different books be arranged on a bookshelf?

We have 8 choices available for the first spot on the shelf. Choose a book out and place it on the shelf. Now we have 7 left to choose from for the second spot, etc...

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 5 \cdot 3 \cdot 2 \cdot 1 = 8! = 40,320$$

We find $8!$ (on the calculator, enter 8, then MATH button. Go to the PRB menu. Enter 4 or scroll down to the ! sign. Then enter again for the answer)

But this is also called a PERMUTATION, $P(8,8)$, the permutation of 8 things taken 8 at a time.

What if we arranged only 3 of the books? How many ways to do this?

$8 \cdot 7 \cdot 6 = 336 = \frac{8!}{5!} = \frac{8!}{(8-3)!}$

This is a permutation of 8 things taken 3 at a time, or $P(8,3)$. You can find these on your calculator as $8nPr3$. (enter 8 on the home screen. hit the MATH button then choose the PRB menu. enter 2 or scroll down to nPr and enter. then hit 3 then enter again)

The general formula for the permutation of n things taken r at a time is

$$P(n,r) = \frac{n!}{(n-r)!}$$

If some of the objects being arranged are the same, then we have to find those permutations that look different, DISTINGUISHABLE PERMUTATIONS.

Say we have a total of 6 marbles. 3 of them are blue, 2 of them are green and 1 is red. How many different distinguishable permutations are there?

The formula for the number of distinguishable permutations of N items where n_1 items are of type 1 and n_2 items are of type 2 and n_r items are of type r is

$$\frac{N!}{n_1!n_2!\ldots n_r!}$$

So in our example we would have

$$\frac{6!}{3!2!1!} = 60$$

We often only want a group or subset of items from a finite set, not an arrangement. When the order of the objects doesn't matter, it is a COMBINATION.

Say we wanted to choose 3 of 8 books to lend to a friend. The order they are chosen won't matter, just if the books are chosen to be in the group or not. We start by finding the number of ways to arrange the 3 of 8 books and then divide by the number of ways the 3 books can be arranged among themselves (as they are all still in the same group). We have

$$\frac{8 \cdot 7 \cdot 6}{3!} = 56 = C(8,3) = \frac{P(8,3)}{3!} = \binom{8}{3}$$

In general, the COMBINATION of n things taken r at at time is

$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Example - A researcher has 12 plants. 5 of them are wheat, 4 are corn and 3 are rye plants. A sample of 3 plants in chosen.

a) How many different samples of 3 plants are there?

We have 12 plants and we pick a group of 3 to be our sample,

$$C(12,3) = \binom{12}{3} = 12nCr3 = 220$$

b) How many different samples have 2 wheat plants?

We will choose 2 from the 5 wheats and the remaining 1 from the 7 that are not wheat:

$$C(5,2) \cdot C(7,1) = \binom{5}{2} \cdot \binom{7}{1}$$

$$= (5nCr2) \cdot (7nCr1) = 10 \cdot 7 = 70$$

c) How many different samples will have one of each kind?

$$C(5,1) \cdot C(4,1) \cdot C(3,1) = \binom{5}{1} \cdot \binom{4}{1} \cdot \binom{3}{1}$$

$$= (5nCr1) \cdot (4nCr1) \cdot (3nCr1) = 5 \cdot 4 \cdot 3 = 60$$

d) How many samples will have at least one corn?

First find which samples will satisfy our "at least" statement:

1 corn and 2 not corn + 2 corn and 1 not corn + 3 corn and 0 not corn

Now work out how many ways to make each sample and then add them all up:

1 corn and 2 not corn is

$$C(4,1) \cdot C(8,2) = \binom{4}{1} \cdot \binom{8}{2} = (4nCr1) \cdot (8nCr2) = 4 \cdot 28 = 112$$

2 corn and 1 not corn is

$$C(4,2) \cdot C(8,1) = \binom{4}{2} \cdot \binom{8}{1} = (4nCr2) \cdot (8nCr1) = 6 \cdot 8 = 48$$

3 corn and 1 not corn is

$$C(4,3) \cdot C(8,0) = \binom{4}{3} \cdot \binom{8}{0} = (4nCr3) \cdot (8nCr0) = 4 \cdot 1 = 4$$

So we will have

$$112 + 48 + 4 = 164$$

ways to choose a sample with at least one corn.

An alternative way to solve this problem is to realize that in any sample of 3 plants we can have $3C, 0C^c$ or $2C, 1C^c$ or $1C, 2C^c$ or $0C, 3C^c$. So we can find the number of ways to choose without restrictions (220) and subract our one missing case, $0C, 3C^c$,

$$C(4,0) \cdot C(8,3) = \binom{4}{0} \cdot \binom{8}{3} = (4nCr0) \cdot (8nCr3) = 1 \cdot 56 = 56$$

And so find the number of ways to have at least one corn is 220 - 56 = 164, same as calculating the three cases that work. This is is a useful technique for "at least one" type problems.

Example - How many ways can a hand of 5 cards have three cards of the same suit?

Example - In a bag of 10 apples there are 3 bad, apples.

How many ways can a sample of 4 be chosen?

How many ways in which

a) None are rotten?

b) 1 is rotten?

c) 2 are rotten?

d) 3 are rotten?

e) 4 are rotten?

Example - A minivan can hold 7 passengers. An adult must sit in one of the two front seats and anyone can sit in the rear 5 seats. A group of 4 adults and 3 children are to be seated in the van. How many different seating arrangements are possible?

Example - How many different two item pizzas are possible if there are 8 different toppings and doubles are allowed?

Example - You have a class of 20 children, 10 boys and 10 girls. How many ways can the children be seated in a row if boys and girls must alternate?

Example - You take a mutiple choice test with 3 questions and each question has 5 possible answers. How many ways can the test be answered?

An EXPERIMENT is any activity with an observable result. Tossing a coin, rolling a die or choosing a card are all considered experiments. An OUTCOME (or SAMPLE POINT) is the result of a the experiment. The set of all possible outcomes or sample points of an experiment is called the SAMPLE SPACE.

Flip a coin, $S = \{H, T\}$

Roll a 6-sided die, $S = \{1, 2, 3, 4, 5, 6\}$

Choose a card and note the suit, $S = \{\clubsuit, \spadesuit, \diamondsuit, \heartsuit\} = \{C, S, D, H\}$.

We always want our sample space to be UNIFORM. That is, each of the outcomes in the sample space has an equally likely chance of happening. In our sample spaces for the coin we assumed it was a fair coin and then the two outcomes will have the same chance of happening.

Example - You have a cup with the scrabble tiles W, X, Y, Z in it. You choose one tile at random. What is the sample space? Is it uniform?

Example - You have a cup with the tiles A, A, B, C. You choose one at random. What is the sample space? Is it uniform?

Example - From a bin of 6 apples (4 red and 2 green) a sample of 2 is chosen. What is the sample space? Is it uniform?

An EVENT is a subset of the sample space.

Example: List all events possible when a coin is tossed.

$\emptyset$ (event that the coin is a head AND a tail)

$\{H\}$ (event that the coin is a head)

$\{T\}$ (event that the coin is a tail)

$\{H, T\}$ (event that the coin is a head OR a tail)

So there are 4 events from our sample space of 2 outcomes!

Example - a bag has one red, one blue and one green marble. A single marble is chosen from the bag.

a) What is the sample space?

b) What events are possible?

Example - A coin is tossed and the side noted and a card is drawn and the color noted.

a) What is the sample space?

b) What events are possible?

To find the theoretical probability of an event occuring we must first find a UNIFORM SAMPLE SPACE (the outcomes are all equally likely). If there are n outcomes in the sample space, they will each have a probability of $1/n$ of occuring. The outcomes are MUTUALLY EXCLUSIVE - that is, only one can occur during the experiment.

We want to arrange the outcomes (also called simple events) in a probability distribution table:

outcome	probability
$\{s_1\}$	$1/n$
$\{s_1\}$	$1/n$
$\vdots$	$\vdots$
$\{s_1\}$	$1/n$

We say $P(\{s_1\}) = P(s_1) = P_1 = 1/n$

Probability distribution tables have the following properties:

1. $0 \leq P(s_i) \leq 1$
2. $P(s_1) + P(s_2) + ... + P(s_n) = 1$
3. $P(\{s_i \cup s_j\}) = P(s_i) + P(s_j), i \neq j$

Example - A card is chosen from a standard 52 card deck and the suit is noted. Find the probability distribution table for this experiment:

outcome	probability
$\heartsuit$	1/4
$\spadesuit$	1/4
$\clubsuit$	1/4
$\diamondsuit$	1/4

What is the probability that the card is red ($\heartsuit$ OR $\diamondsuit$)?

$P(red) = 1/4 + 1/4 = 2/4$

Example - a family has two children. Find a probability distribution table for the number of girls in the family.

Example - From a bin of 6 apples (4 red and 2 green) a sample of 2 is chosen. What is the probability distribution table for the number of green apples in the sample?

The empirical probability of an outcome is determined by the relative frequency it occurs. You can find the relative frequency by doing an experiment.

Example - A survey was done of students for how many earrings they are wearing. The following results were found:

no. of earrings	no. of students	relative frequency
0	100	100/345 = .29
1	60	60/345 = .17
2	120	120/345 = .35
3	30	30/345 = .09
4	20	20/345 = .06
5 or more	15	15/345 = .04
total	345	1

So we can find the probability that a person has 3 or more earrings on as

$P(x > 3) = P(3) + P(4) + P(\text{5 or more})$

$= 30/345 + 20/345 + 15/345 = .19$