CHAPTER 7: PROBABILITY

Basics of Probability

An EXPERIMENT is an activity with an observable result.

Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the OUTCOME or SAMPLE POINT.

So the two possible outcomes from tossing a coin H T

The set of all outcomes or sample points is called the SAMPLE SPACE of the experiment.

$S = \{H, T\}$

An EVENT is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

If a set has n members, there are 2^n subsets.

Consider tossing a coin. The sample space is S = {H,T}. $2^2 = 4$ SUBSETS

The events that are possible in this experiment are

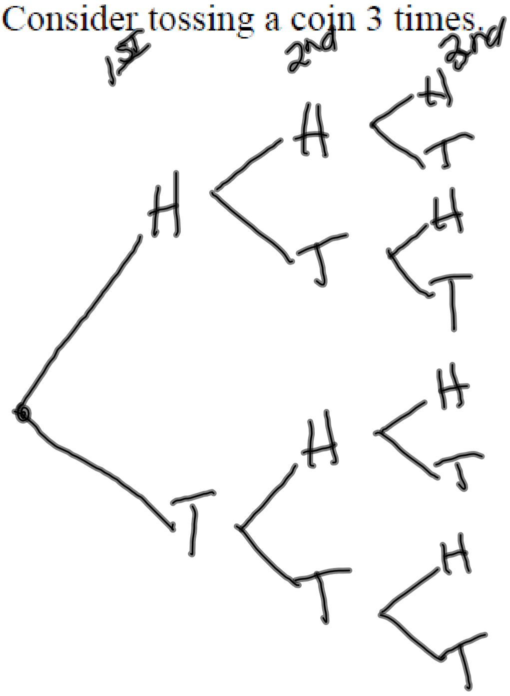
$\emptyset, \{H\}, \{T\}, \{H,T\}$

When 2 outcomes are in the sample space, there are 4 different events [subsets].

$S = \{1, 2, 3, 4, 5, 6\}$

If a 6-sided die is rolled, the sample space is

Consider tossing a coin 3 times.



HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

$2 \cdot 2 \cdot 2 = 8$

$S = \{HHH, HHT \qquad \}$

These sample spaces are all UNIFORM.

A sample space in which each of the outcomes has the same chance of occurring is called a UNIFORM SAMPLE SPACE.

A uniform sample space has equally likely outcomes.

A non-uniform sample space for tossing a coin three times is

$N = \{3H0T, 2H1T, 1H2T, 0H3T\}$

The uniform sample space is

$S = \{HHH, HHT, \ldots\ TTT\}$ $n(S) = 8$

$2^8 = 256$

Find the event E where E = {x|x has exactly one head}

$E = \{HTT, THT, TTH\}$

Find the event E where E = {x|x has two or more heads}

$E = \{HHT, HTH, THH, HHH\}$

Find the event E where E={x|x has more than 3 heads} $\emptyset$

Rolling two fair six-sided dice. $S = \{$

$\underset{1^{st}}{6} \cdot \underset{2^{nd}}{6}$

$= 36$

1~1,	2~1,	3~1,	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6 $\}$

These sample spaces have all been finite. That is, we can list all the elements.

An infinite sample space has to be described; you can't list all the elements:

What is the sample space for the time spent working on a homework set?

$S = \{ t \mid t \geq 0, \text{ t in minutes} \}$

Describe the event of spending between one and two hours on a homework set.

60 min 120 min

$E = \{ t \mid 60 < t < 120 \}$

Definition of Probability

When we toss a fair coin the two outcomes in the sample space $S = \{H, T\}$ are equally likely, so the probability of each outcome is 1/2.

$$P(\{H\}) = \frac{1}{2} = P(H)$$

This is a THEORETICAL PROBABILITY based on the sample space having equally likely outcomes.

In general, this is the way we will find probability, by using a sample space of EQUALLY LIKELY OUTCOMES.

The probability of an event, P(E) is a number between 0 and 1, inclusive

We can also calculate the EMPIRICAL PROBABILITY of an event by doing an experiment many times.

For example, you could toss a coin and note how many times it comes up heads (shown in book)

or you could roll a die and count how many times a 1 is rolled.

number of tosses (m)	number of 1's rolled (n)	relative frequency (n/m)
20	3	3/20 = .15
100	19	19/100 = .19
1000	152	152/1000 = .152

$1/6 = .1666666$

UNIFORM SAMPLE SPACE $S=\{s_1, s_2, \ldots, s_n\}$,

$\{s_1\}, \{s_2\} \ldots \{s_n\}$ are called SIMPLE events

Notice these simple events are MUTUALLY EXCLUSIVE as only one can occur.

PROBABILITY DISTRIBUTION TABLE:

Event	probability
S_1	$1/n$
S_2	$1/n$
$\vdots$	
S_n	$1/n$

Properties of probability distribution tables:

$0 \le P(S_i) \le 1$

$P(S_1) + P(S_2) + \ldots P(S_n) = 1$

$P(S_i \cup S_j) = P(S_i) + P(S_j)$

Toss a coin three times. There are 8 equally likely outcomes:

Event	Probability
HHH	1/8
HHT	1/8
⋮	⋮
TTT	1/8

Find the probability distribution table for the $\boxed{\text{number of heads}}$ when a coin is tossed 3 times.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Event	Prob
0H	1/8
1H	3/8
2H	3/8
3H	1/8

$$\frac{3}{8} + \frac{1}{8} = \frac{4}{8} \left(= \frac{1}{2}\right)$$

What is the probability of 2 or more heads?

Suppose the instructor of a class polled the students about the number of hours spent per week studying math during the previous week. The results were 69 students studied two hours or less, 128 students studied more than two hours but 4 or less hours, 68 students studied more than 4 hours but less than or equal to 6 hours, 30 students studied more than 6 hours but less than or equal to 8 hours and 14 students studied more than 8 hours.

t = # of hours spent studying

Arrange this information into a PDT and find the probability that a student studied more than 4 hours per week

$$\frac{68}{309} + \frac{30}{309} + \frac{14}{309} = \frac{112}{309}$$

Event	# of students	Prob
$0 \le t \le 2$	69	69/309
$2 < t \le 4$	128	128/309
$4 < t \le 6$	68	68/309
$6 < t \le 8$	30	30/309
$t > 8$	14	14/309
	309	

What is the probability of rolling a sum 2 or a sum of 12 using two fair die?

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$$\frac{1}{36} + \frac{1}{36} = \frac{2}{36}$$

What is the probability of rolling a sum of 7?

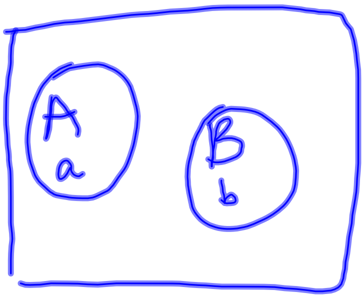
1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$$\frac{6}{36}$$

Rules for Probability

If event A and event B are mutually exclusive then

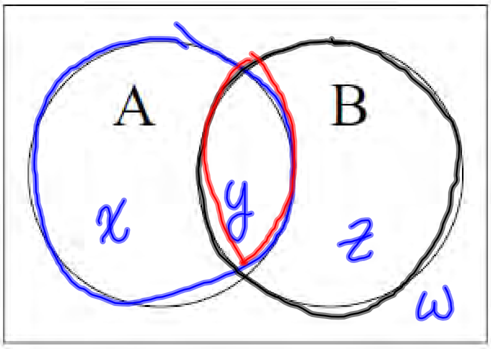
$P(A \cup B) = P(A) + P(B)$



In general, A and B have some outcomes in common so we have the union rule for probability:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$x + y + z + w = 1$



$$P(E \cup F) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

Example: Let

E={x|x is a sum of 7} $\Rightarrow$ 6/36

and

F={x|x is a 6 on the green die} = 6/36

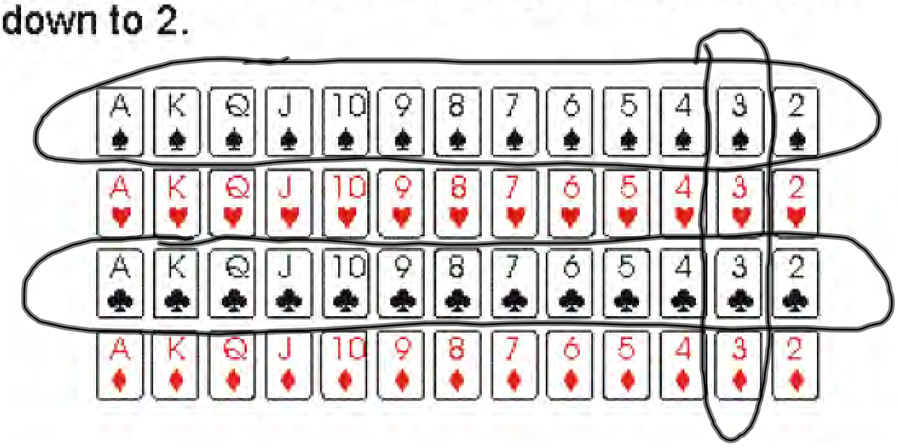
1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$\frac{11}{36}$

$P(E \cap F) = 1/36$

What is the probability that you have a sum of 7 OR a 6 on the green die?

A standard deck of 52 cards has 4 suits, each with 13 cards. The suits are spades, ♠, hearts, ♥, clubs, ♣ ,and diamonds, ♦. The cards in each suit are numbered from Ace, King, Queen, Jack, ten down to 2.



Example - If a single card is drawn from a standard deck of cards, what are the probabilities of

$$P(B \cup 3) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

a) a 9 or a 10? Let

$$\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

b) a black card or a 3?

$$P(A) + P(A^c) = 1$$

Example - a survey gave the following results: 45% of the people surveyed drank diet drinks (D) and 25% drank diet drinks and exercised ($D \cap E$) and 24% did not exercise and did not drink diet drinks ($D^c \cap E^c$). Find the probability that:

a) a person does not drink diet drinks (D^c). $1 - .45 = .55 = 55\%$

b) does not exercise and drinks diet drinks ($E^c \cap D$). $.2 = 20\%$

c) exercises and does not drink diet drinks ($E \cap D^c$). $= .31 = 31\%$