Last Name: _____ First Name: _____ Section 141-____ UIN: _____

Binomial Probability

For a probability experiment to be binomial, it must have all of the following properties:

- **The number of trials in the experiment is fixed. This value is called *n*.**
- **There are only two outcomes: Success and Failure.**
- **The probability of success in each trial is the same. This value is called *p*.**
- **The trials are independent of each other.**

Part I

1. In a large city 30% of the residents like broccoli (*B*) and the rest do not like broccoli (*N*). Three residents are chosen at random and asked if they like broccoli or not.

a. Are there a fixed number of trials in this experiment? ____
If yes, how many? n = ____

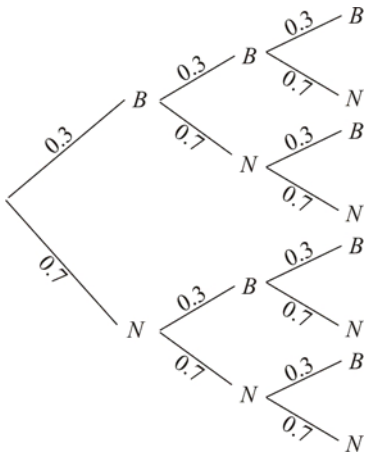
b. Are there only two outcomes in every trial? _____
If yes, what are they? _____ and _____

c. Is the probability of success in each trial the same? _____
If yes, a success is considered to be _____ and
the probability of this occurring is p =_____

d. Does the outcome on one trial change the probabilities of the outcomes on other trials? _____
If no, each trial is said to be _____ of each other.

e. Is this a binomial experiment? _____

2. The tree diagram shows this experiment. How do the properties of binomial experiments relate to this tree diagram?



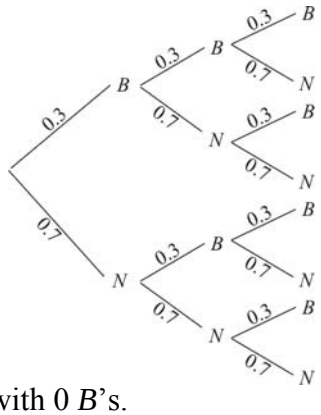
a. There are _____ "stages" in the tree, which correspond to _____.

b. Each "node" has _____ branches that correspond to _____.

c. The probabilities on the tree DO or DO NOT change ~~(circle one)~~ on each pair of branches throughout the tree, which means that _____ _____.

Hint: Your answers to **a-c** should match your answers to **1.**

3. Let X = # of successes = # of residents who like broccoli.
Find the following probabilities using the tree diagram and the product rule.



a. Calculate the probability that 0 residents like broccoli, or $P(X = 0)$.
Note: There is exactly one path on which 0 people like broccoli, i.e, 0 successes.

$$P(X=0) = (0.70)\cdot(0.70)\cdot(0.70) = (0.70)^3$$
$$= 1\cdot(0.30)^0\cdot(0.70)^3$$
$$= C(3,0)\cdot(0.30)^0\cdot(0.70)^3 = \_\_\_\_\_$$

Note: The term $C(3,0) = 1$ represents the number of ways to get 0 B's in 3 trials—the number of paths with 0 B's.
The term $(0.30)^0$ represents traveling along 0 branches that have a 0.30 probability.
The term $(0.70)^3$ represents traveling along 3 branches that have a 0.70 probability.

b. Calculate the probability that exactly 1 resident likes broccoli, i.e., 1 success.

i. What are the three paths on the tree that have exactly 1 B and 2 N's? _____, _____, _____
ii.

$$P(X=1) = (0.30)\cdot(0.70)\cdot(0.70) + (0.70)\cdot(0.30)\cdot(0.70) + (0.70)\cdot(0.70)\cdot(0.30)$$
$$= (0.30)(0.70)^2 + (0.30)(0.70)^2 + (0.30)(0.70)^2$$
$$= \_\_\_\cdot(0.30)^{\_\_}\cdot(0.70)^{\_\_} = C(3,\_\_\_)\cdot(0.30)^{\_\_}\cdot(0.70)^{\_\_} = \_\_\_\_\_$$

c. Calculate the probability that exactly 2 residents like broccoli, i.e., 2 successes.

$$P(X=2) = C(3,\_\_\_)\cdot(0.30)^{\_\_}\cdot(0.70)^{\_\_} = \_\_\_\_\_$$

d. Calculate the probability that all 3 residents like broccoli, i.e., 3 successes.

$$P(X=3) = \_\_\_\_\_$$

4. Organize this information in a probability distribution table with X = # of residents who like broccoli.

X	0	1	2	3
$P(X)$	_____	_____	_____	_____

The probability of r successes in a binomial experiment with n trials and probability of success p is

$$P(X=r) = C(n,r)\,p^r(1-p)^{n-r}$$

5. On your calculator, enter the X values into list L1 and $P(X)$ into list L2. Do 1-var stats L1, L2.

Mean = μ = _____

Standard deviation = σ = _____

Part II

The binomial experiment in **Part I** had the following properties:

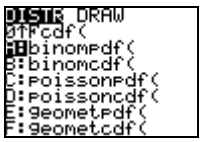
X = # of successes = # of residents who like broccoli, $n = 3$, $p = 0.30$ and $r = 0, 1, 2,$ or 3.

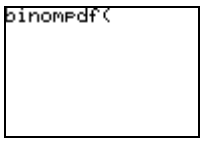
With this information and your calculator, no tree or formula is needed to calculate probabilities.

$$P(X = r) = \text{binompdf}(n, p, r)$$

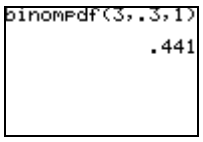
Choose the value of r based on the specifics of the question you are asked.

i. Go to DISTR (above the VARS button) → **ii.** Scroll down to the binomial commands → **iii**. Press ENTER for the **binompdf** command





Now enter the values for n, p, and r. Close the parentheses and press ENTER to evaluate. **Round your answers to 4 decimal places, if necessary.**

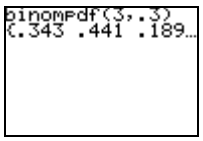


For $r = 1$, the value of $P(X = 1)$ would be found as shown to the right:

1. What is the command to find the probability that exactly 2 residents like broccoli?

$P(X = 2)$ = binompdf(____, ____, ____) = _____ This is the same value as **Part I, 3c.**

If you simply enter n and p in binompdf, it will list the probabilities for every value of X (from $X = 0$ to $X = n$). Use the right and left arrows to scroll through the list.



For the remaining questions, ***assume that 20 residents are surveyed*** from the same city as in **Part I**.

2. What is the probability that exactly 4 residents like broccoli?

a. Success = likes broccoli, $n = 20$, p = _____, r = _____

b. $P(X = 4)$ = binompdf(____, ____, ____) = _____

3. What is the probability that 3 or fewer residents like broccoli?

a. r = desired # of successes = 0, ___ , ___, ___.

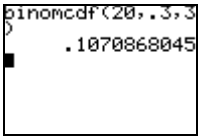
b. $P(0 \le X \le 3) = P(X = 0)$ + _____ +_____ +_____

= binompdf(____, ____, ____) + _____(____, ____, ____)

+ _____(____, ____, ____) + _____(____, ____, ____)

= _____

The calculator command **binomcdf** sums the probabilities of 0 to r successes for you.



$$P(0 \le X \le r) = \text{binomcdf}(n, p, r)$$

The binomcdf command is found under the binompdf command in the DISTR menu.

c. Find the same probability found in **b** using the binomcdf command.

$P(0 \le X \le 3)$ = binomcdf(____, ____, ____) = _____

4. What is the probability that more than 8 residents like broccoli? *Hint:* Refer to the histogram and mark the rectangles you must include.

$r = 9$, ____, … , ____

Although the desired number of successes does not begin at 0, the *complement* of the desired probability is r=0 to r=8 successes, which *can* be calculated with a binomcdf command.

$P(X > 8) = 1 - P(0 \le X \le 8) = 1 -$ binomcdf(____ , ____ , ____) = _____

5. What is the probability that more than 4, but fewer than 15 residents, like broccoli?

$r = 5$, ____, … , ____

This probability can be found by taking the probability of r=0 to r=14 successes and subtracting off the part of this probability that you do not want, r=0 to r=4 successes.

$P(5 \le X \le 14) = P(0 \le X \le 14) - P(0 \le X \le 4)$

= binomcdf(____, ____, ____) – _____(____, ____, ____) = _____

6. What is the probability that between 6 and 12 residents like broccoli?

Hint: The word between means *strictly* inside of the two endpoints.

$r =$ ____, ____, … , ____

$P($____ $\le X \le$ ____$) = P(0 \le X \le$ _____$) - P(0 \le X \le$ _____$)$

= _____

= _____

7. What is the mean number of the 20 surveyed residents who like broccoli? $\mu =$ _____

Standard deviation? $\sigma =$ _____